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The Improvement of the Rheological Model of Leather

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***Abstract*–Fibrous capillary-porous materials, such as woven materials and leather, used in the light industry for clothing and footwear, differ sharply from metallic materials. These differences manifest themselves in a complex relationship between stress and strain, which depends on the strain rate and loading time. A method for determining the identified new rheological parameter of the inert resistance of a capillary-porous material (for example, leather) of accelerated deformation is presented in the article; it consists in determining the indentation force of a conical indenter at a constant speed and the coefficient that considers shear zones, and the angle formed by the boundaries of the material deformation zone and application of force value. An equation for an improved rheological model of leather is derived; it consists of the Kelvin model, the Bingham-Shvedov model, and the Khusanov model, expressed in terms of the rheological parameter in the form of a deformation inertness coefficient. The inclusion of the model of deformation inertness in the form of a coefficient into the rheological model of leather will allow the mathematical description of the rheological parameters of leather and the development of engineering methods for their calculation. The implementation of the developed method will allow obtaining numerical values of the rheological parameter of the inert resistance of the capillary-porous material under its accelerated deformation, namely, a new property of the capillary-porous material, which must be taken into account in various technological processing operations, for example, when treating moisture-saturated leather.**

***Keywords*— Capillary-porous material – leather; rheological model; deformation inertness; deformation; rheological equation.**

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I. INTRODUCTION

In rheology, four intermediate models are known. Elastic model (Hooke's); viscous fluid model (Newton's); plasticity model (Saint-Venant's) and the deformation inertness model. In the elastic model (a spring), the energy spent on the deformation is accumulated and can be returned under unloading. According to Newton's model, a viscous fluid is characterized by the fact that its stresses are proportional to the strain rate. The Saint-Venant plasticity model can be represented as a dry friction element consisting of a plate and a triangle pressed against each other. The Saint-Venant model will not begin to deform until the shear stresses exceed some critical value – the yield strength (ultimate shear stress), after which the element can move at different velocities.

Rheological or structural-mechanical properties characterize the behavior of a material under stress conditions and make it possible to relate strains, stresses, and strain rates to each other under pressure application.

Consider the work of scientists in the study of the deformation properties of leather and other materials, considering the increase in their physical, mechanical, quality and other indices.

The studies by Tournier and Lado [1] and Tournier [2] consider the factors influencing the performance of technological operations, which can contribute to the quality of the collagen fibers of leather after its treatment. The authors claim that their results on the study of leather properties can be applied to various types of leather. In Wright et al. [3], various hide defects that occur while keeping cattle were studied. Studies have shown that the main hide defects appear during the fattening period of cattle. The defects subsequently affect the quality of leather processing and its cost. Automatic classification of visual leather images was developed by Aslam et al. [4]. A set of numerous images of leather was compiled. This classification serves to determine their regularity on leather surfaces. The results showed a 92% accuracy in determining the roughness of the leather surface. In Tournier [5], the effect of liquid treatment on the subsequent mechanical properties of leather was studied. The study results make it possible to determine the strength properties of the collagen fibers of leather. In Liu, Chen, and Latona [6], studies were carried out to assess leather's qualitative and mechanical parameters based on a new ultrasound method. The study's results showed the possibility of evaluating the leather using an ultrasonic wave and determine the characteristics of leather's stretching, stiffness, elongation, and viscosity characteristics. In Van der Merwe [7], the qualitative indices of sheepskin leather of various breeds were studied. At the same time, the tensile strength indices of leather after their liquid treatment were determined.

To improve social, sanitary, and hygienic conditions for workers applying coatings on leather, an automated process of dyeing the surface of the hides with a minimum participation of human factor was proposed by Gouthem et al. [8]. In di Capaci et al. [9], software for modeling and controlling the color quality of dyed leather was developed for robotic carousel equipment. The program allows the required color tone selection for the leather from the database of identified colorimetric models. In Memon et al. [10], the indices of leather bending rigidity and thickness were studied for a change in the drapery coefficient of sheepskin, and their relationship was shown. The results of the study contribute to the selection of the most suitable material for sheepskin garments.

A detailed literature analysis was performed in Dwivedi, Agrawal, and Madaan [11] to determine the aspects of the sustainable manufacturing of leather production. The authors have developed an integrated methodology for identifying and analyzing priority indices to improve the efficiency of leather production. The methodology for analyzing the relationship of priority indices will contribute to implementing an effective program for the development of the leather industry. The study by Thangavelu, Gunasekar, and Jyotishi [12] proposes a hypothesis of the innovation contribution effect on the country's economic development. The results of the feedback analysis based on regression models according to the international innovation index of 154 countries for 2013–2017 were presented. The results showed a significant impact of innovation on income in countries with a high level of development. In Serweta et al. [13], the aging process of footwear leather was studied based on the measurement of rheological properties when determining the degree of its stretching. To determine the shrinkage, footwear leather samples were subjected to mechanical tests to determine their resistance to the aging process. Compared to calfskin, there was a higher resistance of leather to the aging process in cattle and nubuck leather.

Wiśniowski, Skrzypaszek, and Małachowski [14] provides a methodology for selecting a rational rheological model for drilling fluid based on the well-known models of Newton, Bingham, Casson, Ostwald de Wael, and Herschel-Bulkley, Vom Berg and Hahn-Eyring. The authors have developed a computer program called Rheosolution for the automated determination of a rational rheological model for drilling fluid. In Alexe et al. [15], the researchers used polymers in the form of nanocomposites to coat the surface of sheepskin by sputtering. The study showed that the surface of sheepskin treated with nanocomposite materials had improved physical and mechanical properties compared to untreated material samples. In Šobak et al. [16], polymeric films' viscoelastic and mechanical properties were experimentally determined. The parameters of tensile stress and deformation of a coating of thin-film materials were determined on a tensile machine.

In Benmakhlouf et al. [17], the hydro-rheological behavior of leather under its mechanical deformation was experimentally studied. Modeling of changes in temperature and humidity of a leather sample cut from bovine hide under various conditions of its convective drying was developed. It was determined that in order to obtain a quality product, it was necessary to reduce the temperature. The authors of that work stated that this simulation could be applied to leather, reducing the drying time and reaching the moisture content needed. Bahadirov et al. [18], the authors of this article, experimentally studied the technological process of squeezing out excess moisture from wet semi-finished leather products. The influence of the number of layers of wet leather semi-finished products and the feed rate to the processing zone under the pressure of rotating squeezing rollers was determined. The technological process of squeezing the moisture from wet semi-finished leather products was conducted by the vertical feed of wet semi-finished leather products on a base plate between rotating squeezing rollers. At the same time, the productivity of the pressing process was more than five times higher than that of similar roller machines. Amanov et al. [19], the authors of the article, have experimentally determined in laboratory conditions the deformation properties of a semi-finished leather product after its squeezing at a moisture content of 60%. The leather samples from the *Wet-blue* bovine were studied according to their topographic sections: butt, belly, and shoulder, from hides bred in the regions of Uzbekistan. Mathematical dependences of the semi-finished leather product deformation on the pressing force in the butt, belly, and shoulder topographic sections were obtained. It was stated that in the initial loading section, the deformation of samples of the *Wet-blue* semi-finished leather product increases, and then, after its sufficient compaction, the dependence takes on a linear character. The results make it possible to more accurately select the operating parameters of pressure when processing a semi-finished leather product with knife spiral rollers.

A method for soaking hides was developed in Danylkovych, Korotych, and Romaniuk [20]. According to this method, the hide processed in an electrochemically activated water solution turned to leather with improved elastic-plastic properties, in contrast to the existing technology, and contributed to a 2.5% increase in the leather yield area. In Roh [21], a study was made on leather's physical and mechanical properties after punching. The most rational parameters of leather after punching were determined. In the study, different leather samples were perforated at different intervals and evaluated based on their density, softness, coefficient of friction, temperature effects, and mechanical properties. In Eun and Lee [22], the mechanical properties of six different samples of artificial leather were studied. The study showed that an increase in the content of hydroxyls contributes to an increase in cross-links and, accordingly, the mechanical properties of the studied artificial leather increased. In Duo et al. [23], nanofibers' effect on synthetic leather's structure and properties were studied. The study results showed that the nanofibers were uniformly distributed in all directions of the synthetic leather and increased moisture absorption by 22.3%. In Yuan et al. [24], the process of formaldehyde diffusion in the hidden structure was studied and estimated using the models of Blondeau et al., Ataka et al., and Xiong et al. The study results showed that the Xiong et al. model agrees well with experimental results compared to the other two models. Besides, the model of Xiong et al. is appropriate for calculating the results of studies from the point of view of saving time. In Sathish, Madhan, and Rao [25], a new method of chrome tanning of leather based on polymeric tannin was developed. Experiments were conducted; their results showed that the physical and mechanical properties of the leather processed by the new method coincide with the properties of the leather of traditional chrome tanning. The advantage of the new method is the improvement of the organoleptic parameters of leather. In Kumitsky et al. [26], analytical dependences of the force parameters of the stress-strain state of the pressed composite were obtained, as well as the ratios for the kinematic characteristics of the pressing process and the expression for stress relaxation during the process of holding the material under pressure after the end of active pressing. The study's results make it possible to experimentally determine the numerical values of the dynamic viscosity coefficient and the stress relaxation time, which are important characteristics in controlling the pressing processes.

The Moscow State University of Design and Technology (MGUDT) has developed a new method for evaluating the deformation properties based on computer analysis of the process of material relaxation – its elastic recovery after unloading. Analysis of this process makes it possible to single out three deformation components and calculate six indices that most fully describe the elastic, viscous, and plastic properties of materials that characterize the deformation properties of a material and reflect the mobility of various elements of its internal macro- and microstructure [27]. The " Relax " installation consists of a personal computer, an electromechanical sensor device, and an electronic signal conversion unit. The material to be tested is fixed along the circular contour and loaded centrally with an indenter – a light rod with a tip that can move freely in the vertical direction. The computer program fixes the initial position of the indenter, taking into account the sagging of the sample, the maximum stroke under load, and the steady state after unloading, and calculates the corresponding relative strains and average stresses in the sample [27].

The study of the deformation properties of the leather fabric was conducted by Kucherova [28]. She developed a methodology for evaluating the deformation properties of fur leather fabric and, based on the developed assessment methodology, gave recommendations on identifying such rational modes of fur leather processing technology as temperature, thermal and mechanical influences. She also studied the influence of the basic technological factors of heat, moisture, and mechanical effects on deformation properties. The study of the fur leather fabric was performed by the methods of mathematical statistics using various computer software. She revealed and investigated the shrinkage of the layers under bending and torsion of samples of the fur leather fabric when tested under the influence of heat and moisture. She also developed a method for quantitative assessment of the shrinkage of the layers of fur leather fabric. She developed a rapid method to assess and predict the characteristics of semi-finished fur products. However, in her studies, she did not take into account the deformation inertness of the leather semi-finished product.

The disadvantage of the rheological models and studies considered above of the leather properties lies in the fact that, at the initial time, the deformation of the material occurs instantly. However, this can be accepted for solid materials only. In the materials such as semi-finished leather and leather, where under mechanical processing, the moisture content reaches 45 to 60%, the deformation does not occur instantly but is described by a certain curve. Therefore, the authors have developed a new rheological model of leather that takes into account the rheological parameter of the inert resistance of leather.

II. Materials and Methods

A method to determine a new rheological parameter of the inert resistance of a capillary-porous material [29] (for example, leather) of accelerated deformation is described in this article; it consists in determining the immersion force of a conical indenter at a constant rate and a coefficient that takes into account shear zones and the angle formed by the boundaries of deformation zones of the material and force value application. The research is devoted to studying and determining the rheological parameter of inert resistance under the deformation of a capillary-porous material.

A method developed by the authors to define a new rheological parameter m_l – the inert resistance of a capillary-porous material of accelerated deformation consists in determining the immersion force of a conical indenter at a constant rate and a coefficient that takes into account shear zones, as well as the angle formed by the boundaries of the deformation zone of the capillary-porous material and the force value application on the graph of the velocity gradient along the depth of immersion.

Then, from the graph, we determine the limiting dynamic strain stresses as the conical indenter is introduced, and then we determine the ability of the rheological parameter of the capillary-porous material to accelerate deformation according to the following formula

$$m_{li} = \frac{\tau_{0i} \cdot h_{0i}^2}{u_{0i}^2} \quad (1)$$

where, τ_{0i} – are the limiting dynamic stresses that change as the indenter immerses; h_{0i} is the indenter immersion depth; u_{0i} – is the indenter immersion rate.

The implementation of this method [29] will allow obtaining the numerical values of the rheological parameter ml – the inert resistance of the capillary-porous material of accelerated deformation, namely, a new property of the capillary-porous material, which must be taken into account in various technological processes for processing moisture-saturated capillary-porous materials.

The essential difference is that at the initial stage of material deformation during the introduction of a conical indenter, the rheological parameter m_l is determined, which characterizes the material's properties to the inertial deformation resistance of the material under accelerated deformation. The article by Khusanov [30] is devoted to the study of medium deformability by inertia and their models.

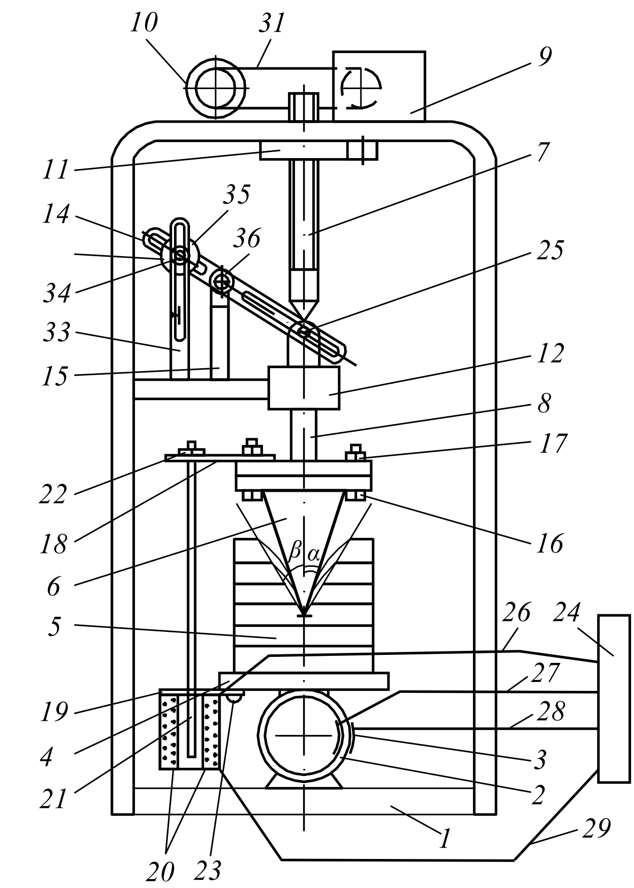

Fig. 1 Scheme of an experimental stand for determining the rheological parameter of a capillary-porous material

The implementation of the method for determining the rheological parameter of the material to inert resistance under accelerated deformation is conducted by a device. Figure 1 shows a diagram of the device, and Fig. 2 shows a graphical definition of the ultimate shear stress and a velocity gradient along the immersion depth of a conic indenter.

The device contains frame 1, on which torque ring 2 with sensor 3 is fixed; table 4, material or medium under study 5, conical indenter 6, screw pair 7, rod 8, variator 9, electric motor 10, drive 11, bracket 12, load 13, lever 14, support 15, bolts 16, nuts 17, plates 18, 19, inductive sensor 20, core 21, nut 22, screw 23, an amplifier with oscilloscope 24, axis 25, wires 26, 27, 28, 29.

The rod 8 moving vertically downwards in the direction of bracket 12 is balanced by means of load 13 and lever 14 hinged on support 15 so that the pressure on the test material 5 from indenter 6 is minimal. The indenter 6 is fixed to rod 8 with bolts 16 and nuts 17.

An inductive displacement sensor 20 is attached to table 4 by means of plate 19, the core 21 of which, ending with an adjusting screw, is connected by means of plate 18 to rod 8 with bolt 16 and nut 17. Axis 25 is fixed to rod 8 and moves along the groove on lever 14.

The operation of the device for the proposed method for determining the rheological parameter m_l – the inert resistance of the material to its accelerated deformation is given below. When conducting measurements in the zone of initial deformation of stress and strain rates, we obtain the curve shown in Fig. 2.

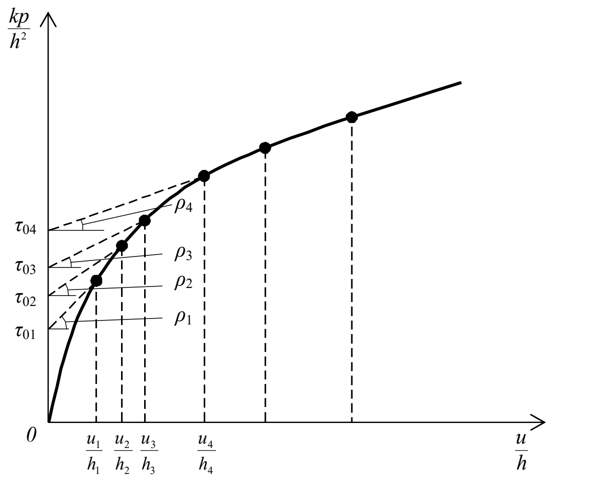

Fig. 2 Graphical determination of the limiting shear stress of a capillary-porous material

Figure 3 shows the dependence graphs of the amount of moisture extracted from a wet capillary-porous material in percent on different feed rates and pressing forces.

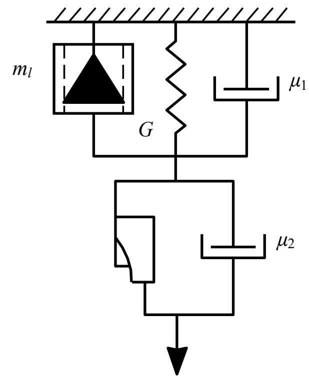

Fig. 3 Improved rheological model of leather

According to the curve in Fig. 2 it follows that the studied material 5 has a solid structure and as the indenter is introduced, the structure of the material is damaged, and hence, the limiting dynamic stress τ_0 changes, taking successively the values τ_{01}, τ_{02}, …, τ_{0i}, determined by drawing tangents at the points of the curved line.

In Khusanov [30], the rheological retardation model of an elastic-viscous-inert material has the form $\tau = f_3 G_1 \gamma + f_i \mu_1 \dot{\gamma} + f_2 m_{li} \ddot{\gamma}$. In a particular case, $f_2 m_{l2} \ddot{\gamma} = \tau_0$ and if we accept $\tau = f_3 G_1 \gamma + f_i \mu_1 \dot{\gamma} + f_2 m_{li} \ddot{\gamma}$, then we have $\tau - \tau_0 = f_i \mu_1 \dot{\gamma}$, and $f_1 + f_2 + f_3 = 1$.

f_1, f_2, f_3 – are the volume fractions of viscous and deformation inert and other properties of materials. G_1 is the modulus of elasticity, μ_1 is the viscosity, m_l is the inert resistance under accelerated deformation of the material; γ, $\dot{\gamma}$,$\ddot{\gamma}$– are the strain, strain rate, strain acceleration, entering the law of an inertly deformed material expressed by$\tau_0 = m_l \ddot{\gamma}$; limited to one-dimensional and stationary cases and written in averaged parameters, we will have $\tau_0 = m_l \frac{u^2}{h^2}$, we substitute $m_l \frac{u^2}{h^2}$ instead of τ_0 into equation $\frac{kp}{h^2} = \tau_0 + k_1 h_n \frac{u}{h}$ and obtain $\frac{kp}{h^2} = k_1 h_n \frac{u}{h} + m_l \frac{u^2}{h^2}$, where p is the force of immersion, $k = \frac{1}{\pi tg\alpha}$, α is the angle at the top of the conical indenter, $k_1 =$

$\frac{2}{tg\beta - tg\alpha}$, β – is the angle formed by the boundaries of the deformation zone during the introduction of the conical indenter. It is determined experimentally from a photograph of the indentation section obtained after the indenter is inserted into the material with colored boundaries of the layer, h is the immersion depth, $h_n = tg\rho / k_1$, ρ– is the slope angle of the limiting shear stress – τ_0.

Thus, we have obtained an equation with the rheological parameter m_l, which determines the ability of the material under study to undergo accelerated deformability. Since $\frac{u_i}{h_i}$, β , and τ_{0i} can be determined experimentally, then, since τ_{0i}, $\frac{u_i^2}{h_i^2}$ are known, by relations $\tau_{01} = m_{l1}\frac{u_1^2}{h_1^2}$, $\tau_{02} = m_{l2}\frac{u_2^2}{h_2^2} \ldots \tau_{0i} = m_{li}\frac{u_i^2}{h_i^2}$, the values of the newly introduced parameter $m_{l1}, m_{l2}, \ldots, m_{li}$ can be determined.

Thus, a new method was developed for determining an important objectively existing rheological parameter m_l – inert resistance under accelerated deformation of the material. An experimental bench for determining the rheological parameter of a capillary-porous material works is shown in Fig. 4.

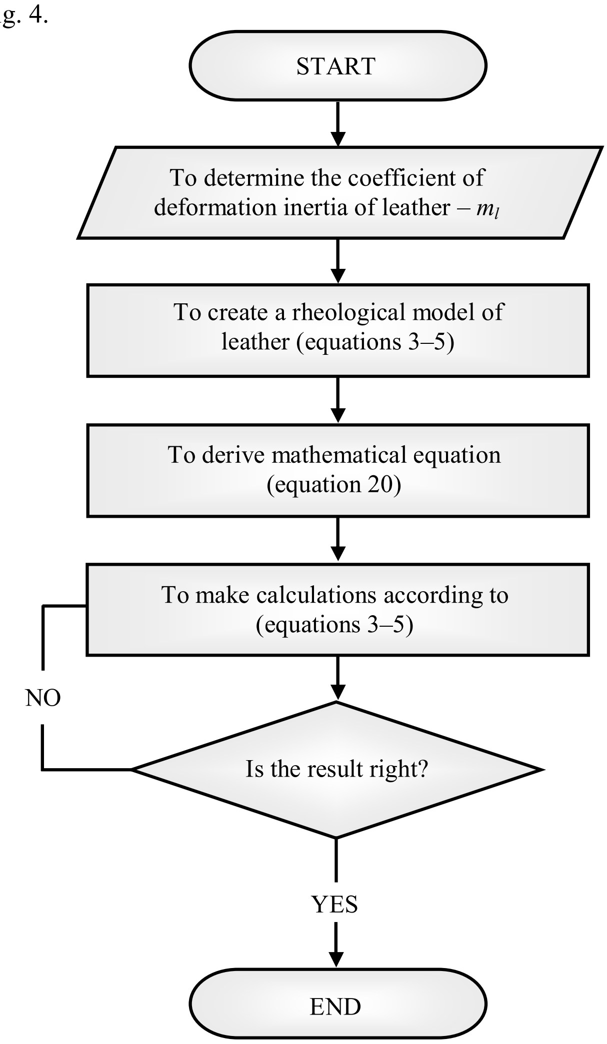

Fig. 4 Block diagram of the proposed method for improving the rheological model of leather

Before the start of the experiment, the weight of rod 8 with conical indenter 6, with plates 18, 19 with induction sensor 20, and with core 21 is balanced by load 13. The strain gauge 3 is preliminarily calibrated using an amplifier with oscilloscope 24. When electric motor 10 is turned on, variator 9 transmits rotation to gear pair 11, which rotates screw pair 7, compresses rod 7, and transfers force to the conical indenter, embedded in the material under study 5.

At the same time, core 21 also moves and determines the immersion depth by induction sensor 20. The pressing force of conical indenter 20 into the material under study 5 is determined by strain gauge 3, displayed on the oscilloscope 24. Then we plot the dependence of the shear stress $\tau_0 = f\left(\frac{u}{h}\right)$, where u is the strain rate, h is the immersion depth. Based on the analysis of existing models and the new model of deformation inertness proposed, an improved rheological model of leather is given, considering the newly revealed phenomenon of deformation inertness of materials. Research methods include the following stages (Fig. 4):

1) Stage 1. Review and critically analyze rheological models of leather and identify their shortcomings. To develop a new, improved rheological model of leather, we reviewed and critically analyzed existing rheological models of leather found in the literature [1–30]. So, it was stated that in all the considered models, the physical parameter characterizing the inert resistance to accelerated deformation of the material, particularly leather, was not considered.

2) Stage. Preparation of the necessary leather materials and selection of samples for research. The leather material and its samples were chosen based on the formulas of mathematical methods of statistics [18–19].

3) Stage. Preparation of equipment and installation of measuring devices and instruments for experiments. Equipment drawings were designed, and a test bench was made, on which strain gauge sensors were mounted, and connected by wires to the corresponding devices.

4) Stage. Calibration of measuring instruments and strain gauges. The strain gauge sensors were calibrated for the pressing force and the conveying speed.

5) Stage. Conducting test experiments. To conduct the experiments, the electric motor was turned on and the force and speed of penetration into the test material, in particular into leather, were applied. The oscilloscope was turned on and its screen showed the pressing force, speed and depth of penetration into the material. Then the angle β was determined; it was formed by the boundaries of the deformation zone of the material under loading of the indenter and the angle at the top of the indenter cone - α, as well as the true calibration data that were inserted into formula (1); then the value of the coefficient of deformation inertness of the material - ml was determined.

6) Stage. Mathematical processing of research results. The measured data from the devices were substituted into the appropriate expression and the results after the calculation were obtained.

7) Stage. Acquisition of experimental data, interpretation of data and their analysis based on mathematical processing. After mathematical processing of the data results obtained, the dependence of the force applied to the indenter on the ratio of the speed and depth of the penetration into the leather material was plotted. Mathematical methods determined the errors in conducting and measuring experimental data.

8) Stage. Final stage. The value of the coefficient of deformation inertia of the material – m_l is determined.

III. Results and Discussion

Consider the fundamental models that can be used in the study of the rheological properties of leather. Four intermediate models of idealized materials are known: Hooke's elastic model; Newton's viscous fluid model; Saint-Venant's plasticity model and the deformation inertness model.

Consider a rheological model that connects in parallel the Hooke model, the Newton model, and a model of an inert deformation body, where the deformations of these three models are equal, and their total stress is equal to the sum of three models of idealized bodies (Hooke's, Newton's and the deformational inertness model).

Consider a rheological model connecting in parallel the Kelvin model and the model of an inert deformation body, where the strains of these two models are equal, and the stresses of these models are equal to their sum.

$$\tau = G\gamma_I + \mu_1\dot{\gamma}_I + m_l\ddot{\gamma}_I.$$

The Shvedov–Bingham model is successively connected to this model. The Shvedov–Bingham model consists of the Saint-Venant model plus, in parallel, the Newton model.

$$\tau = \tau_0 + \mu_2\dot{\gamma}_{II}.$$

Consider a rheological model connected in parallel with the Kelvin model and the model of an inert deformation body, successively connected with the Shvedov–Bingham model (Fig. 3) and obtain a system of two equations that must be coupled into a general rheological equation

$$\begin{cases} \tau = \tau_I \pm \tau_{II} \\ \gamma = \gamma_I + \gamma_{II}. \end{cases} \quad (2)$$

We differentiate once $\gamma = \gamma_I + \gamma_{II}$– the total deformation equal to the sum of the deformation of the first and the second models.

$$\dot{\gamma} = \dot{\gamma}_I + \dot{\gamma}_{II}. \quad (3)$$

We differentiate $\dot{\gamma} = \dot{\gamma}_I + \dot{\gamma}_{II}$ once again and obtain the following expression

$$\ddot{\gamma} = \ddot{\gamma}_I + \ddot{\gamma}_{II} \quad (4)$$

The first rheological equation, which consists of connected in parallel Hooke's, Newton's and deformation inertness models, is

$$\tau = G\gamma_I + \mu_1\dot{\gamma}_I + m_l\ddot{\gamma}_I. \quad (5)$$

The second Shvedov-Bingham rheological equation, which consists of connected in parallel Saint-Venant's and Newton's models, has the following form.

$$\tau = \tau_0 + \mu_2\dot{\gamma}_{II}. \quad (6)$$

We divide equation (5) by G and differentiate once and get the following expression

$$\frac{1}{G}\dot{\tau} = \dot{\gamma}_I + \frac{\mu_1}{G}\ddot{\gamma}_I + \frac{m_l}{G}\dddot{\gamma}_I. \quad (7)$$

We divide equation (6) by μ_2 and obtain the following expression

$$\frac{\tau}{\mu_2} = \frac{\tau_0}{\mu_2} + \dot{\gamma}_{II} \quad (8)$$

We sum up equation (7) with equation (8) and obtain the following expression

$$\frac{\dot{\tau}}{G} + \frac{\tau}{\mu_2} - \frac{\tau_0}{\mu_2} = \dot{\gamma} + \frac{\mu_1}{G}\ddot{\gamma}_I + \frac{m_l}{G}\dddot{\gamma}_I. \quad (9)$$

We differentiate equation (8) once and obtain the expression

$$\frac{\dot{\tau}}{\mu_2} = \ddot{\gamma}_{II}. \quad (10)$$

We multiply equation (9) by $G/\mu_1$and obtain the equation

$$\frac{\dot{\tau}}{\mu_1} + \frac{G}{\mu_1\mu_2}\tau - \frac{G\tau_0}{\mu_2} = \frac{G}{\mu_1}\dot{\gamma} + \ddot{\gamma}_I + \frac{m_l}{G}\dddot{\gamma}_I, \quad (11)$$

$$\frac{\dot{\tau}}{\mu_2} = \ddot{\gamma}_{II}. \quad (12)$$

We sum up the equation (11) and equation (12) and obtain the equation

$$\left(\frac{1}{\mu_1} + \frac{1}{\mu_2}\right)\dot{\tau} + \frac{G}{\mu_1\mu_2}\tau - \frac{G}{\mu_2}\tau_0 = \frac{G}{\mu_1}\dot{\gamma} + \ddot{\gamma}_I + \frac{m_l}{G}\dddot{\gamma}_I \quad (13)$$

$$\left(\frac{\mu_1+\mu_2}{\mu_1\mu_2}\right)\dot{\tau} + \frac{G}{\mu_1\mu_2}(\tau - \tau_0) = \frac{G}{\mu_1}\dot{\gamma} + \ddot{\gamma} + \frac{m_l}{G}\dddot{\gamma} \quad (14)$$

We differentiate equation (12) once and obtain the following equation

$$\frac{\ddot{\tau}}{\mu_2} = \dddot{\gamma}_{II}. \quad (15)$$

We differentiate the equation $\ddot{\gamma} = \ddot{\gamma}_I + \ddot{\gamma}_{II}$ once and obtain the equation

$$\dddot{\gamma} = \dddot{\gamma}_I + \dddot{\gamma}_{II}. \quad (16)$$

Equation (14) is multiplied by μ_1/m_l

$$\left(\frac{\mu_1+\mu_2}{\mu_1\mu_2}\right)\dot{\tau} + \frac{G}{\mu_1\mu_2}(\tau - \tau_0) = \frac{G}{m_l}\dot{\gamma} + \frac{\mu_1}{m_l}\ddot{\gamma} + \dddot{\gamma}\ , \quad (17)$$

$$\frac{\ddot{\tau}}{\mu_2} = \dddot{\gamma}_{II}. \quad (18)$$

We sum up the equation (17) with equation (18) and obtain

$$\frac{1}{\mu_2}\ddot{\tau} + \left(\frac{\mu_1+\mu_2}{m_l\mu_2}\right)\dot{\tau} + \frac{G}{m_l\mu_2}(\tau - \tau_0) = \frac{G}{m_l}\dot{\gamma} + \frac{\mu_1}{m_l}\ddot{\gamma} + \dddot{\gamma}. \quad (19)$$

Multiplying equation (19) by μ_2, we obtain an improved rheological equation for leather that takes into account deformation inertness

$$\ddot{\tau} + \left(\frac{\mu_1+\mu_2}{m_l}\right)\dot{\tau} + \frac{G}{m_l}(\tau - \tau_0) = \frac{\mu_2 G}{m_l}\dot{\gamma} + \frac{\mu_1\mu_2}{m_l}\ddot{\gamma} + \mu_2\dddot{\gamma}, \quad (20)$$

τ – is the normal stress of the leather material; [Pa]
m_l – is the coefficient of deformation inertness of the leather material; [Pa·(s)^2]
μ_1 – is the viscosity of the leather material (at the initial stage of deformation); [Pa·s]
μ_2 – is the viscosity of the leather material (in the subsequent stage of deformation); [Pa·s]
G – is the modulus of elasticity of the leather material; [Pa]

$\dot{\tau}$ – is the rate of change in the stress of the leather material; [Pa/s]
$\ddot{\tau}$ – is the acceleration of change in the stress of the leather material; [Pa/(s)^2]
γ – is the tensile strain of the leather material; [–]
$\dot{\gamma}$ – is the rate of tension of the leather material; [1/s]
$\ddot{\gamma}$ – is the acceleration of tension of the leather material; [1/(s)^2]
$\dddot{\gamma}$ – is the change in the acceleration of tension of the leather material; [1/(s)^3]
τ_0 – is the limiting dynamic stress in the leather structure; [Pa].

In further work, based on the real parameters of leather a numerical solution will be conducted using the rheological model of leather obtained (20). The results of the theoretical studies conducted showed a refinement of the rheological model of leather, which means that in the existing rheological models, the leather is deformed instantly, from a certain value. Unlike the rheological models considered, in the rheological model proposed by the authors, the deformation of leather starts from zero, which is shown in Fig. 2 and experimentally determined on the stand shown in Fig. 1. On the whole, the inclusion of deformation inertness in the form $\tau = m_l\ddot{\gamma}$ to the rheological model of leather allows us to determine the initial parameters for calculating the rheological characteristics of the leather and makes it possible to more accurately describe the process of leather deformation as compared to the considered models of leather.

IV. Conclusion

Thus, an improved rheological model of leather (20) was obtained, which takes into account the deformation inertness expressed in terms of m_l. The improved rheological model consists of the Kelvin model, the Bingham-Shvedov model, and the Khusanov model, expressed by a rheological parameter in the form of coefficient m_l of the deformation inertness of leather. The inclusion of deformation inertness in the form of the coefficient m_l in the rheological model of leather will allow a more accurate mathematical description of the rheological parameters of leather, which, in turn, will allow more accurate development of engineering methods for their calculation and operating parameters of technological machines for mechanical processing of leather. Ultimately, this will improve the quality of the finished product and reveal the reserves for the intensification of technological operations of the mechanical processing of leather.

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