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Mental Action Way of Understanding (WoU) and Way of Thinking (WoT) Students in Statistics Learning in Higher Education

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***Abstract*— This study aims to examine in depth the Mental Action Way of Understanding (WoU) and Way of Thinking (WoT) of students in learning Statistics in Higher Education and the relationship between WoU and WoT. This research is qualitative research using the Grounded Theory approach. The participants involved were 36 people in the city of Bandung, West Java, Indonesia. Validity and reliability using the Krippendorff content analysis method with the Inter-coder Agreement (ICA) test using Krippendorff alpha coefficients. The results showed that there was a significant relationship between students' mental action statistics and students' WoU and WoT in Statistical Understanding and Thinking. The relationship between WoU and WoT: (1) Students' WoU will build such a statistical framework. (2) Students' WoT affects how statistical modeling from their statistical knowledge. This relationship fits the philosophy of statistics and inductive inference. The statistical mental action of WoU students was to make models, analyze, calculate and generalize. The mental actions of Statistics students from WoT were interpreting, analyzing, predicting, reasoning, and inferring. The WoU and WoT mental action models developed the PPDC Cycle model and the Gal's statistical literacy model. the PPDAC Cycle model by Wild and Pfankuch, consists of five dimensions, and one of the dimensions was the PPDAC Cycle. The statistical literacy model by Gal consists of two dimensions, namely the dimensions of statistical knowledge and disposition. The novelty of this research was developing the PPDAC Cycle statistical thinking model by Wild and Pfankuch, and the statistical literacy model on the statistical knowledge dimension by Gal. This study's results can be considered in designing statistical learning in universities and measuring students' understanding and statistical thinking skills.**

***Keywords*— Mental action,;WoU and WoT statistics; grounded theory.**

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I. Introduction

How to understand and think statistically is how students understand and think about statistics based on inductive inference and statistics philosophy. Philosophically, statistics is a science that studies events that occur in the real-world utilizing modeling to be analyzed mathematically to obtain conclusions inductively. Inductive inference is a way in which sample data taken randomly from a population are tested statistically as a population representation. The theory used to test these statistics is probability calculus.

The development of information technology triggers everyone to get fast-paced and valid information, with no exception for statistical information. In reporting their performance results, almost all institutions use statistics because they are more concise and information dense. Therefore, many institutions hold statistical analysis training using statistical analysis software to improve their performance. The need to understand statistical information has been highlighted by various organizations in the United States, including the American Library Association. Despite the increasing need for statistics teaching, many students have historically viewed statistical education as difficult and unpleasant to learn and by many instructors as a frustrating and unprofitable course. As more students enroll in introductory statistics courses, instructors face many challenges in helping these students succeed in their study of statistics [1], [2].

There are some weaknesses among teachers, students, and professors in universities, specifically in the ability to read and interpret data in tables and graphs and create tables, graphs, and reports [3]. New students must use statistical inquiry to introduce complex statistical ideas over time [4] informally, and this experience can provide a solid foundation when formal statistics are introduced.

Understanding statistical information is not just knowing the average, standard deviation, graphs, etc. However, it must have statistical thinking skills, such as understanding how the data is obtained. Statistical thinking involves understanding why and how statistical investigations are conducted and the ideas underlying statistical investigations [5]. The foundation of statistical investigation rests on the assumption that many real situations cannot be assessed without proper data collection and analysis [6]. Statistical thinking includes understanding data, modeling, choosing analytical methods, estimating probabilities, testing hypotheses, making conclusions, understanding the context of problems, and criticizing as well as evaluating the results of statistical analysis. Statistical models must capture elements of real situations; so that the resulting data will lead to the context of statistical knowledge. The topics that can be taught to students for Students' Statistical Thinking and Reasoning are the topic of data distribution, the concept of Average, concepts of Variance and Covariance, Normal Distribution, Sample and Sample Distribution [7], [8].

Some problems in Statistics learning are summarized by Tishkovskaya and Lancaster, namely: How to apply Statistics content in problem-solving, learning is carried out by teachers who do not understand Statistics, anxiety and lack of interest in learning Statistics, difficulties for students of other disciplines in studying probability theory and statistics, weak knowledge of basic statistics and mathematical reasoning[9].

Several models and learning designs (didactic design) Statistics can be an alternative to the above problems: Game-based learning for the topic of opportunity[10], [11], a project-based learning model on the topic of average scores[12]. The didactic model and design of Statistics should be built on the historical and philosophical foundations of how Statistics became a science (epistemology) and how Statistics is applied to learning (Axiology). Therefore, it is necessary to study the statistical way of thinking based on these historical and philosophical foundations in-depth.

Fig. 1 is The statistical thinking model promoted by Wild and Pfankuch offers a statistical thinking model in the empirical investigation [13]. Meanwhile, in this study, the statistical thinking model is examined from how students understand descriptive statistics and inference statistics and what mental actions arise from statistical thinking and understanding statistics. According to Harel[14], the way of thinking (WoT) and the way of understanding (WoU) is one part of the mental action (Mental Act) of a person's cognitive products. The purpose of this study was to examine in depth the statistical mental actions of students from the way of thinking (WoT) and how to understand (WoU) student statistics in statistics learning in universities.

The PPDAC cycle is concerned with abstracting and solving statistical problems that are based on a larger "real" problem. This model was used to examine WoU and WoT students in learning statistics because this model adopts the framework of statisticians thinking in finding statistical knowledge. Epistemologically, how statisticians obtain statistical knowledge is important as a reference in statistical learning.

The purpose of this study was to examine in-depth Mental Action Way of Understand (WoU) and Way of Thinking (WoT) students in Statistics learning in Higher Education. Novelties in this study were a design of statistical learning and measuring students' statistical understanding and thinking skills: interpreting, modeling, analyzing, calculating, generalizing, reasoning, estimating, and concluding.

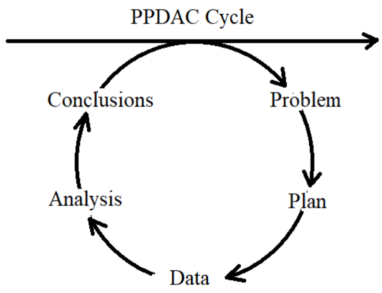


Fig. 1 PPADC Cycle framework for statistical thinking in empirical inquiry

A. *Philosophical and Historical Statistics*

Statistics is a branch of applied science of how humans obtain information on natural and social phenomena to be used as recommendations for making wise decisions. Statisticians use the inductive inference method in generalizing sample data, such as Ronald Fisher, Jerzy Neyman, and Bruno de Finetti [15], [16].

Historically, the development of statistics has emerged since the 17th century. This development arose from the philosophical questions of statistical scientists *"How does someone make conclusions about unobserved data based on observed data?"* Bandyopadhyay and Forster, Philosophically and historically, argue that until now, there are four different paradigms of statisticians, namely; *Classical statistics/Error Statistics, Bayesian Statistics, Likelihood-based Statistics, and the Akaikean-Information Criterion (AIC)-Based Statistics*[16]. Fig. 2 explains how statistical data can estimate parameters? Statistical data is the size of the sample and the parameter is the size of the population.

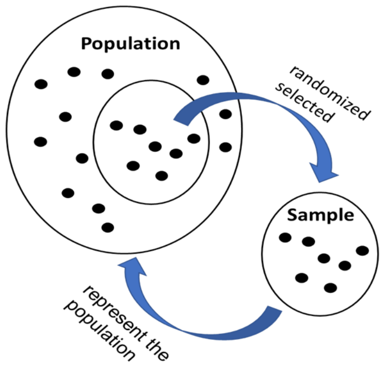


Fig. 2 Generalize the sample to the population

Classical statistics use frequency statistics as an important key to understand the significance test, while Bayesian statistics use conditional probability as the central principle in hypothesis testing. Likelihood-based statistics stand between Bayesianism and Frequentists. Likelihoodists argue that one must interpret the likelihood function to understand what the data itself says. While AIC-based statistics answer questions about curve fitting, and how the model can capture the signal behind the noise. AIC assumes that there is a probability distribution that produces independent data points, even though the actual probability distribution is unknown.

B. Descriptive Statistics and Inference Statistics

Descriptive statistics is a way to describe and summarize a set of data which includes: frequency tables and graphs, central phenomena (mean, median, mode), distribution phenomena (variance, standard deviation, etc.), data characteristics, and sampling techniques[17]. The part of statistics that deals with the method of concluding inductively are called inferential statistics, which includes: the distribution of random variables, parameter estimation (point estimation and interval estimation), and hypothesis testing (Parametric Statistics and Non-Parametric Statistics).

Table 1 is an example of a statistical description of the economic growth of ASEAN countries from 2012 to 2016. The data in table 1 can be summarized into a summary of the data in Table 2. The data are examples of descriptive Statistics.

TABLE I
ECONOMIC GROWTH OF ASEAN COUNTRIES IN 2012 – 2016

No	Country	Year				
		2012	2013	2014	2015	2016
1	Brunei Darussalam	0.91	-2.13	-2.51	-0.41	-3.17
2	Cambodia	7.31	7.43	7.07	7.04	7.02
3	Indonesia	6.03	5.56	5.01	4.88	5.02
4	Laos PDR	7.90	7.97	7.98	7.45	6.94
5	Malaysia	5.47	4.69	6.01	4.97	4.24
6	Myanwar	7.33	8.43	7.99	7.29	6.30
7	Philippines	6.68	7.06	6.22	5.91	6.84
8	Singapore	3.87	5.00	3.57	1.93	2.00
9	Thailand	7.24	2.73	0.92	2.94	3.23
10	Vietnam	5.25	5.42	5.98	6.68	6.21

TABLE II
DATA SUMMARY OF ECONOMIC GROWTH OF ASEAN COUNTRIES IN 2012-2016

Variable	N*	Mean	St.Dev.	Min	Med	Max
Brunei Darussalam	0	-1.462	1.673	-3.170	-2.130	0.910
Cambodia	0	7.1740	0.1847	7.0200	7.0700	7.4300
Indonesia	0	5.300	0.485	4.880	5.020	6.030
Laos PDR	0	7.648	0.452	6.940	7.900	7.980
Malaysia	0	5.076	0.687	4.240	4.970	6.010
Myanmar	0	7.468	0.808	6.300	7.330	8.430
Philippines	0	6.542	0.469	5.910	6.680	7.060
Singapore	0	3.274	1.309	1.930	3.570	5.000
Thailand	0	3.41	2.32	0.92	2.94	7.24
Vietnam	0	5.908	0.584	5.250	5.980	6.680

In descriptive statistics learning, students must be able to read and interpret data in text and context. They must also be able to calculate statistical formulas and read as well as interpret statistical symbols that are part of descriptive statistics. In line with the development of computer technology, statistical software needs to be understood by students as a tool to make it easier to process and analyze statistics. Based on the statistical description in table 1, we can predict the economic growth of each country in the next five years, which is one part of statistical inference[18].

Table 3 is another example of statistical inference, where we want to know the statistical literacy of 114 undergraduate students of the Faculty of Education in one of the Education Universities in Indonesia. He used statistical one-tail hypothesis testing to conclude the statistical literacy of all students in Indonesia[19].

TABLE III
MEAN AND STANDARD DEVIATIONS OF STATISTICAL LITERACY

Dimension	overall			Score		95%	
	Mean	SD	Level	Max	Min	t	p
Statistical literacy	12.33	2.17	Mod.	16	8	1.62	0.054
Knowledge component	10.87	1.98	Mod.	15	6	2.00	0.024
Literacy Skill	1.75	0.67	Mod.	3	1	0.80	0.214
Statistics knowledge	2.80	0.96	Mod.	4	1	3.34	0.001
Mathematics knowledge	1.80	0.71	Mod.	3	0	4.51	0.000
Context knowledge	1.43	0.72	Mod.	2	0	0.44	0.329
Critical Question	3.09	1.21	Mod.	5	1	0.79	0.214
Disposition component	1.46	0.71	Mod.	2	0	0.90	0.184
Belief Attitudes	0.64	0.58	Mod.	1	0	0.74	0.232
Critical Stance	0.82	0.48	Low	2	0	1.56	0.061

Statistical test stages: 1. Establish statistical hypotheses (H_0) and research hypotheses (H_1), 2. Choose a significance level α, 3. Compare z_{count} with z_{table} (t_{count} with t_{table}), 4. decide to accept or reject H_0, 5. Conclude. Before setting H_0 and H_1, we must determine whether the statistical test uses a one-tail or two-tails test. Then, we must be able to explain the meaning of the results of hypothesis testing so that readers who are unfamiliar with statistics can understand the results of their research.

II. MATERIALS AND METHODS

This study is qualitative research using the Grounded Theory (GT) approach, where the ultimate goal of GT is the discovery of theory. The number of participants involved in the study was 36 participants in Mathematics Education at one of the universities in Indonesia. Collecting data using an open questionnaire, Observation (Learning Video), and open interviews. Data analysis used open coding, axial coding, and selective coding. The validity and reliability of the data used Krippendorf content analysis, namely the Inter-Codes Agreement (ICA) test using Krippendorf alpha coefficients.

Fig 3 shows 3 general stages in qualitative research using the GT method: open coding, axial coding, and selective coding. The three stages are detailed by us in the research procedure using the flowchart diagram.

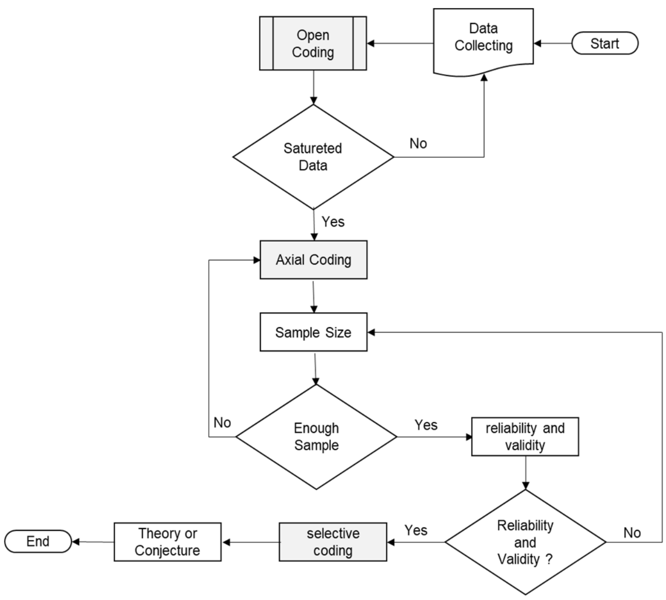


Fig. 3 Flowchart of Research Methods and Procedures GT

A. Data Collecting

Stages of data collection and analysis: First, participants were asked open-ended questions (questionnaires) about how they understood statistical information from various reliable sources[20]. Second, participants were divided into small groups to discuss solving problems consisting of 4 topics: 1. Data and Sampling Techniques, 2. Sample and population meaning, Sample distribution, and point and interval estimation, 3. Distribution of random variables and the relationship between Empirical and Theoretical Means, 4. Normal Distribution and probabilities. Third, we conducted an open interview with each group to explore how to think and understand the statistics of the participants after solving the given problem. Data analysis is carried out simultaneously with data collection, meaning that data analysis does not have to wait for saturated data because further data collection depends on the results of data analysis whether the data is sufficient (saturated) or not.

Fig. 4 was a problem in topic 1 to stimulate participants in how to mental action of WoU and WoT they understand and think about data and sampling techniques. Participants were asked three questions about the problem of traditional markets. If you were a researcher, what did you study about traditional markets? What data should be taken in the study and the sampling technique? State the types and characteristics of the data you will take!

Problem 1

Sumber: http://hipni.blogspot.co.id/2016/01/definisi-kegiatan-ekonomi-menurut-para.html

The picture beside is a traditional market that sells various kinds of basic food needs.
Suppose you are a researcher.

a. What will you research?
b. What sample data will be taken for the research?
c. Define the characteristics and types of data taken in the study!

Fig. 4 The problem in topic 1

Fig. 5 was a problem in topic 2 to stimulate participants in how to mental action of WoU and WoT they understand and think about Sample and population meaning, Sample distribution, and Point and interval estimation. Participants were given the task of estimating parameters (point estimation and confidence interval) based on a sample of survey results and testing the hypothesis of whether the researcher's claims were true. They also understand and think about samples and populations and how samples are generalized to populations.

Problem 2

In a 2014 survey by the Directorate of fishing business services, a random sample of 700 respondents was taken. From the results of data analysis, the average community satisfaction index (CSI) is 69.87 good criteria. Assume the standard deviation of the entire population served is 15.5. Define:

a. What is the CSI population average for the entire community with a 95% significance level?
b. The researcher claims that the CSI of all the communities served is correct 69.87. Use the 99% confidence level to prove the researcher's claim!
c. Use the 99% confidence level, check for CSI mean 55, 60, and 65 respectively correct? What statements can you conclude?

Fig. 5 The problem in topic 2

Fig. 6 was a problem in topic 3 to stimulate participants in how to mental action of WoU and WoT so they understand and think about the distribution of random variables and the relationship between Empirical and Theoretical Means. Participants were given crime census data from the Police Crime Control Bureau in 2016, and they were assigned to perform simulations to find the distribution of random variables and how the sample average estimates the population means.

The number of crimes according to the Statistical Data of the Indonesian Police Operations Control Bureau in 2016.

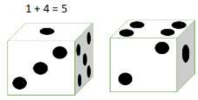
No	Province	Number of Crimes	No	Province	Number of Crimes
1	Jakarta	44.401	17	Papua	7.194
2	East Java	35.437	18	South Kalimantan	6.809
3	North Sumatra	35.248	19	East Nusa Tenggara	6.709
4	West Java	27.805	20	West Kalimantan	6.669
5	South Sumatra	20.575	21	West Nusa Tenggara	6.015
6	West Sumatra	16.277	22	Bali	5.032
7	South Sulawesi	16.088	23	Banten	5.002
8	Central Java	15.958	24	Riau islands	4.892
9	Jambi	10.564	25	Bengkulu	4.463
10	D I Yogyakarta	9.092	26	Southeast Sulawesi	3.655
11	Riau	9.595	27	Gorontalo	3.372
12	Lampung	9.218	28	Central Kalimantan	2.681
13	Central Sulawesi	8.988	29	Bangka Belitung Islands	1.875
14	East Kalimantan	8.764	30	Maluku	1.943
15	D I Aceh	8.048	31	West Papua	1.356
16	North Sulawesi	7.837	32	Nort maluku	814

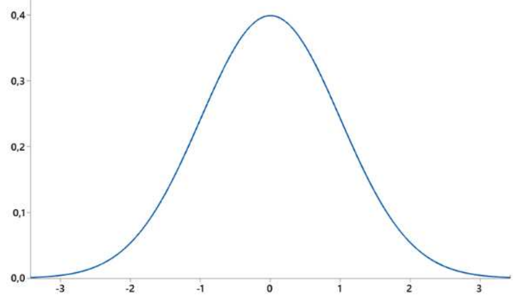
a. Calculate Mean total crime in Indonesia in 2016
b. Make a simple randomly selected data simulation of 5 samples with different sizes, then calculate the average of each.
c. Repeat the above work with 10,000 repetitions
Estimated population mean using the above simulation, what do you think about it.

Fig. 6 The problem in topic 3

Fig. 7 was a problem in topic 4 to stimulate participants how in mental action of WoU and WoT, so they understand and think about Normal Distribution and probabilities. Participants were given a graph of the probability density function. They must describe the significance, critical points, critical regions, and probabilities.

Example
1 + 4 = 5





Experiment with throwing two dice at the same time, counting the number of circles that appear.

a. Create a frequency distribution table for the probability that the number of dice will appear.
b. Graph the probability distribution function of the experiment.
c. Use simulation using statistical tools to replicate the sample 1000 times. Compare this with the probability density function of the standard normal distribution.

Fig. 7 The problem in topic 4

B. Open Coding

Data collection with open coding was carried out simultaneously until the data was saturated. The code is the abstract idea of the participant and the researcher because the researcher and the participant were subjects in qualitative research. Codes are obtained from sentences, one or several paragraphs, and the researcher uses the Atlas.ti program in open coding.

C. Axial Coding

Axial coding was performed after all codes appeared repeatedly and no new codes appeared. In axial coding, the researcher connects all the categories and sub-categories, and the researcher focuses on the core categories. The core category is the category to which all categories are connected.

D. Test the Validity and Reliability of the sample

The researcher took code samples to conduct the ICA test to verify that the code made by the researcher was no different from other coders. The term sample in qualitative research differs from that in quantitative research. The sample is the number of codes selected in the semantic domain, while the population is the total number of codes in the semantic domain after being merged (total code). The semantic domain is many sentences in the data or documents cited by the researcher when conducting open coding. In each quote, there can be more than one code which is called Multi-valued Coding. While the semantic domain consists of one or more quotes. The sample size using the formula is in equation (1)

$$N_c = 2 \cdot z_\rho^2 \left(\frac{(1+\alpha_{min})(3-\alpha_{min})}{4(1-\alpha_{min})P_c(1+P_c)} - \alpha_{min} \right) \quad (1)$$

While the formula calculated the Krippendorff ICA coefficient is in equation (2)

$$\alpha = 1 - \frac{D_o}{D_e} \quad (2)$$

D_o is the observation of agreement, and D_e is the expectation of disagreement.

E. Selective Coding

The final stage of the GT method is selective coding. At this stage, the researcher focuses on the relationships of each category and the memos that the researcher noted. Researchers examine these relationships in-depth to obtain a theory or conjecture.

III. Results and Discussion

Open questionnaires are generally used in qualitative research by qualitative analysis [21]-[22]. This is to maintain the researcher's intervention so that it interferes with the validity of the data. There are four questions asked to participants in the open questionnaire, namely:

- How do you understand statistical information from various reliable sources?
- What skills must be possessed to understand statistical information?
- Is there any statistical information on social media that should be criticized? Why?
- How do you interpret and explain that statistical data has validity and reliability?

The results of open coding and axial coding of the questionnaire and interview data were 155 codes which the researchers reduced to 96 codes. 8 codes often appear, namely: mental action, WoU, WoT, Descriptive Statistics, Inference Statistics, mathematical reasoning, generalizing, and estimating.

Fig. 8 shows topic 1 of data and sampling techniques, the statistical way of thinking of the participants is to define the data based on the research objectives, then they use sampling techniques to obtain data to be analyzed into statistics.

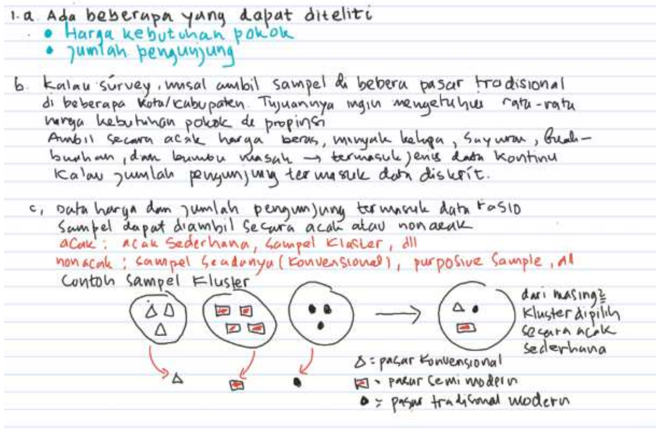


Fig. 8 Defining Variables and Sampling Techniques Planned by Participants

They made symbols and picture illustrations for the planned sampling technique. While in the second topic (Problem 2), participants defined random variables as functions that map a set of people to real numbers and illustrate them with pictures. This activity demonstrates the mental action of statistical modeling, analyzing, and interpreting data[13]. Figure 9 is the answer to one of the small discussion groups. The translations are as follows:

- There are several that can be researched, including prices of basic necessities and the number of visitors.
- If the survey, take a sample from several traditional markets in the city and district. The aim is to find out the average cost of goods sold in the province. Randomly take the price of rice, coconut oil, vegetables, fruits, and cooking spices -> including the type of continuous data. The number of visitors includes discrete data.
- Data on price and number of visitors, including ratio data. Samples can be taken randomly or non-randomly. random sample: simple random, cluster sample, etc. Not random: improvised (conventional) samples, purposive samples, etc.

Fig. 9 shows topic 2 about Sample and population meaning, Sample distribution, and point and interval estimation. Participants used the standard normal distribution function approach to estimate the population mean by the sample mean; with different levels of significance, participants concluded that the length of the confidence interval is based on the length of the margin of error, that the longer the margin of error, the more estimated the average population value will be in the confidence interval. The margin of error depends on the value of the selected confidence interval. The error margin length is directly proportional to the confidence interval. Even in this topic, participants understand the sample as part of the population. They understand that statistics (sample) can estimate parameters (population). This activity demonstrates the mental action of inferring, calculating, estimating, and reasoning[13].

Figure 9 is one of the answers to the small discussion group, *a. from population N taken as many as 200 people who are illustrated in the picture. Sampling technique using a simple random technique, b. from the 90% confidence level, alpha 1% is obtained. Use the standard normal distribution graph then compare it with the results of the empirical calculations.* H_0:: *the average community satisfaction index is 70.* H_1: *the average community satisfaction index is not 70. Based on the results of calculations using the standard normal distribution, the calculated z value is in the area where* H_0 *was accepted. This means that the researcher's claim is true: the average community satisfaction index is 70. c. confidence intervals with variations in the average value can be concluded depending on the margin of error so that the margin of error is extended for an average of 5.60 and 65.*

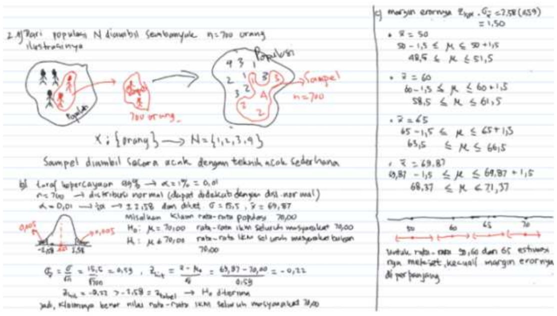


Fig. 9 Defining random variables and making Conclusions about Interval Estimation

Table 4 and Fig. 10 show topic 3 about the distribution of random variables and the relationship between Empirical and theoretical Means. Participants understand the sample and population from the simulation of census data (population) by taking several samples of various sizes. Simulations were carried out with 100, 200, 300, and 400 repetitions[7]. Table 4 was the result of 100 random sampling by participants. They concluded that the populations were almost the same as the formula in equation (3). Then they resampled with 1000 repetitions to produce the graph in Figure 10, which was compared with the standard normal probability density function. The student's statistical way of thinking is estimating parameters by statistics using a relative frequency distribution (probability density function). Participants describe the relative frequency distribution using excel software. The flow of students' statistical thinking in estimating the average value of the population by the average value of the sample is first a random sampling technique using random numbers in Excel and making a relative frequency distribution table, then calculating the average using the formula $x{\cdot}\mathrm{P}(x)$ so that the average value is almost close to the population mean value. Finally, they concluded that the population means could be approximated by the sample mean. They found a statistical formula in equation (3) to estimate the population means based on the sample mean. This activity demonstrates the mental action of inductive inference and reasoning[16].

$$\mu = E(x) = \sum xP(x) \qquad (3)$$

Fig. 10 shows topic 4 about Normal Distribution and probabilities. The student's statistical way of thinking was an inductive inference using a graph sketch of the standard normal distribution function that compares the theoretical alpha value with the empirical alpha value (p-value). Participants represented a cumulative density function with the area under the curve. They showed and concluded that with different levels of confidence, the results of hypothesis testing could be differently indicated by choosing a different alpha value and shifting the critical region depending on the value chosen. The greater the value, the more critical region shifts to the left for the one-tailed test right. This activity demonstrates the mental action of inductive inference, reasoning, and modeling[13].

Next, the researcher conducted an open interview on how the participants understood the statistics of the given problem and how they thought of the solution. What competencies must be possessed to understand the problem and its solution? They say that learning statistics must be contextual because of the statistics of real-world problems, and it would be more interesting if learning statistics were related to the real world and how to apply it to be more understandable. Some of the participants' answers from the interviews are:

Statistics is an applied science; from problems in the real world a statistical model is made. And, from hypothesis testing, conclusions are obtained to answer solutions from the real world. Therefore, we must understand the context of the real problem.

To understand statistical problems, one must understand Descriptive Statistics and Inference Statistics. Whereas. The skills that must be possessed are modeling, reading and interpreting, calculating, analyzing, and concluding, and statistical inference is done inductively.

TABLE IV
PARTICIPANTS CONDUCTED A RANDOM SAMPLING EXPERIMENT 100 TIMES WITH RETURNS.

sample size n = 100											
No	**Province**	***x***	***f***	***p*(*x*)**	***xp*(*x*)**	**No**	**Province**	***x***	***f***	**p(x)**	***xp*(*x*)**
1	Jakarta	44.401	4	0,04	1776	17	Papua	7.194	3	0,03	215,8
2	East Java	35.437	4	0,04	1417	18	South Kalimantan	6.809	3	0,03	204,3
3	North Sumatra	35.248	3	0,03	1057	19	East Nusa Tenggara	6.709	4	0,04	268,4
4	West Java	27.805	2	0,02	556,1	20	West Kalimantan	6.669	3	0,03	200,1
5	South Sumatra	20.575	1	0,01	205,8	21	West Nusa Tenggara	6.015	2	0,02	120,3
6	West Sumatra	16.277	2	0,02	325,5	22	Bali	5.032	2	0,02	100,6
7	South Sulawesi	16.088	3	0,03	482,6	23	Banten	5.002	5	0,05	250,1
8	Central Java	15.958	1	0,01	159,6	24	Riau islands	4.892	1	0,01	48,92
9	Jambi	10.564	4	0,04	422,6	25	Bengkulu	4.463	3	0,03	133,9
10	D I Yogyakarta	9.092	3	0,03	272,8	26	Southeast Sulawesi	3.655	5	0,05	182,8

11	Riau	9.595	4	0,04	383,8	27	Gorontalo	3.372	1	0,01	33,72
12	Lampung	9.218	3	0,03	276,5	28	Central Kalimantan	2.681	1	0,01	26,81
13	Central Sulawesi	8.988	7	0,07	629,2	29	Bangka Belitung Islands	1.875	1	0,01	18,75
14	East Kalimantan	8.764	4	0,04	350,6	30	Maluku	1.943	2	0,02	38,86
15	D I Aceh	8.048	8	0,08	643,8	31	West Papua	1.356	5	0,05	67,8
16	North Sulawesi	7.837	2	0,02	156,7	32	North Maluku	814	4	0,04	32,56
								11.012	100	1	11.060

$$\mu = 11.012 \approx 11.060 = \sum xp(x)$$

x = Numbers of crimes, $p(x)$ = probability

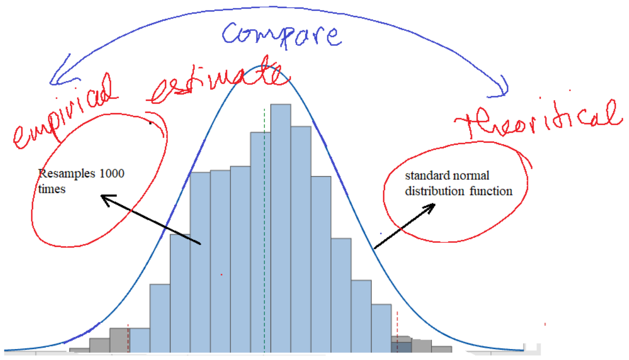


Fig. 10 Resampling 1000 repetitions, Estimating Parameters by Statistics

Table 5 was the result of the participant's work representing the experiment of throwing two dice into mathematics. The mental action of the activity shows the students' WoU, namely modeling. Based on the experimental table of throwing two dice, they made a probability distribution table, where the variable is the number of possible two dice that appear using the counting rules. The mental action in this WoU event was to calculate and analyze. Computing data was the introduction to statistical concepts[6], [23].

Table 6 was the result of calculating and analyzing the discrete random variable x into a probability distribution function. They used statistical tools to obtain the graphs in Figures 11 and 12. The graph in Fig. 11 is a graph of the probability distribution function of the variable x, and the graph in Fig. 12 is a graph of the probability distribution function of the resampling result with 1000 repetitions[24].

TABLE V

PARTICIPANTS MAKE A TABLE OF THE RESULTS OF THE EXPERIMENT OF THROWING TWO DICE SIMULTANEOUSLY

	1	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

TABLE VI

THE PROBABILITY DISTRIBUTION FUNCTION TABLE

x	f	p(x)
2	1	1/36
3	2	1/18
4	3	1/12
5	4	1/9
6	5	5/36
7	6	1/6
8	5	5/36
9	4	1/9
10	3	1/12
11	2	1/18
12	1	1/36
Sum	36	1

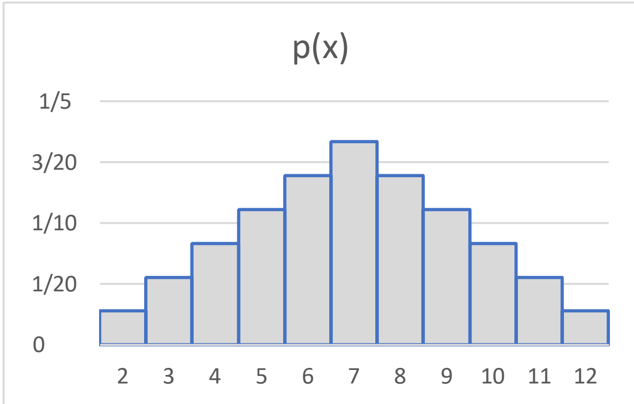


Fig. 11 The probability density function of the trial of two dice

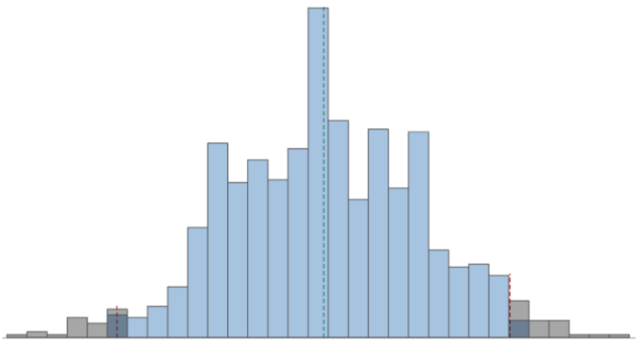


Fig. 12 Resampling 1000 repetitions

Based on the phenomenon of student statistical thinking above, codes (abstract ideas) and the connection between these codes can be made. Researchers focus on codes, and students' Mental Action (Mental Act) of statistical thinking (WoT) and statistical understanding (WoU). From the results of open coding and axial coding, it was obtained that Statistical Thinking and Statistical Knowledge were the core categories. The causal conditions of the core categories are Statistical Reasoning, Mathematical Reasoning, Conclusions, Inductive Thinking, Modeling, Interpreting, Generalizing Samples, Analyzing, and Estimating. Model of learning has the potential to improve students' performance in mental action[25], [26].

Validity and Reliability Test using interpretive concepts Krippendorff alpha coefficient [27]. Data analysis in open coding uses content analysis, while axial coding (to see the relationship between core categories and categories and the relationship between code and data sources) uses constant comparative analysis. The reliability test used the Inter-codes Agreement (ICA) alpha Krippendorff test. The results of the validity and reliability test showed that the coding made by the researcher was valid, and the correlation coefficient between the different coders was quite high $\alpha = 0{,}963$.

In the selective coding stage, Fig. 13 shows the relationship between the core categories and their categories became more stable, and the phenomena that occurred seemed more abstract to building theories about students' statistical thinking[28]. The results of the reliability and validity of the Krippendotff content analysis show a significantly strong relationship between students' statistical thinking (WoT) and the way they understand statistics (WoU) which is interdependent and has implications for statistical mental actions.

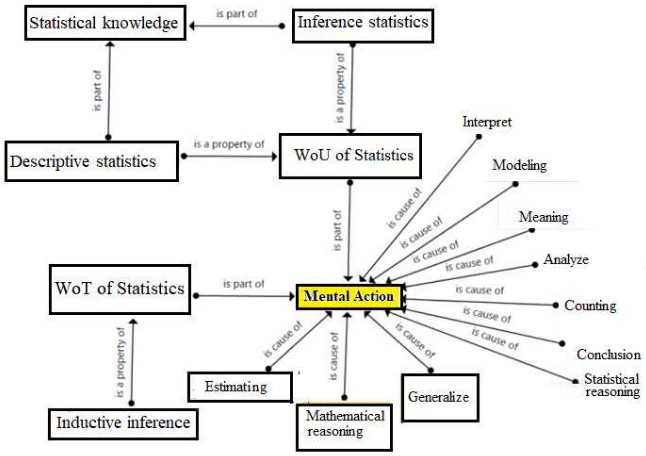
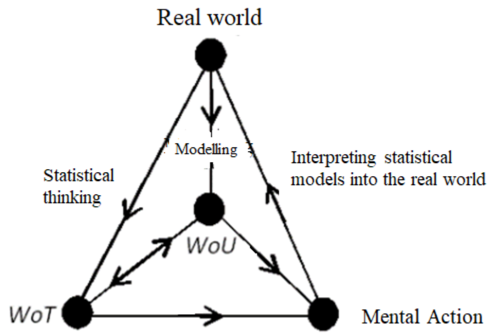
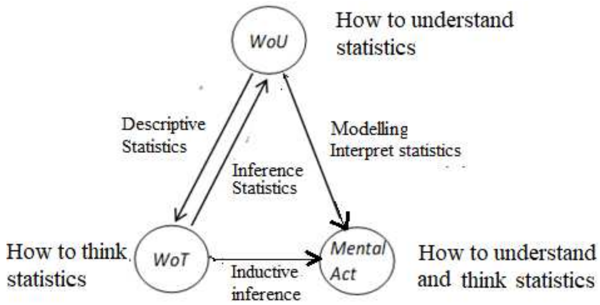


Fig. 13 The selective coding result, Mental action as a core category.

Fig. 14 shows the Mental Statistical Actions model from students' statistical understanding and thinking: interpreting, modeling, analyzing, calculating, generalizing, reasoning, estimating, and concluding. Understanding student statistics depends on statistical knowledge, including descriptive statistics and inference statistics [29]. Meanwhile, students' statistical thinking depends on their inductive thinking ability[21], [30]. Students' ability of statistical knowledge and inductive thinking affects their ability to model statistics from the real world and interpret them in the real world [5], [23]. This model is different from that proposed by previous researchers and Wild and Pfannkuch's model development[13]. At the problem stage: students must interpret and define real problems into statistical models. In this case, students must have statistical data modeling skills[31], [32]. Before planning and analyzing data, students should define the objectives of statistical analysis. The work from modeling to analysis is the student's WoU mental action. In the mental action of WoU, students must understand descriptive statistics well, which is part of statistical knowledge. The statistical test stage was used to conclude. Students must be able to communicate statistical test results into real problems, understand parameter estimation by statistics, understand sample generalization to the population, and think inductively[33-37]. The work from making hypotheses to concluding is the mental action of a student's WoT.



(a) The Mental Action of WoU and WoT Relates to the Real World



(b) Mental Action Model of Student's WoU and WoT

Fig. 14. (a) Relationship of Statistics with the Real World, (b) Mental Action Statistics of WoU and WoT Students.

IV. CONCLUSION

There was a significant relationship between students' mental action statistics and students' WoU and WoT in Statistical Understanding and Thinking. The student's WoU of statistics was by defining the problem into a statistical model: defining random variables, determining sampling techniques, and calculating. The student's WoT of statistics was by statistical test, estimating, determining hypotheses and appropriate statistical tests, concluding, and recommending. According to Wild and Pfannkuch, the plot was included in the dimension of The Investigative Cycle. While statistical thinking (WoT) students use inductive thinking (generalizing hypotheses) according to the statistical inductive inference method. The mental action of WoU and WoT has implications for students' ability to interpret statistics in the real world.

The relationship between WoU and WoT: first, students' WoU will build such a statistical framework. Second, students' WoT affects how to model statistics from their statistical knowledge. The mental actions of WoU and WoT students (the ability to understand statistical concepts and rules, and inductive thinking) are defining, explaining, calculating, estimating, predicting, and generalizing.

Philosophically and historically, Statistics is a science that deals with the real world and use inductive inference methods. Therefore, the statistical way of thinking should use inductive thinking. A decision theory is needed to arrive at inductive inference (point estimation, interval estimation, or hypothesis testing). The decision theory used depends on the theory used by the researcher, whether using Bayesian Theory or Frequency Theory.

The initial skills that students must have are the ability to do sampling techniques and define random variables, understand statistical terms and symbols, and calculate as well as interpret statistics. Statistical thinking can be used to understand the relationship between statistical knowledge and probability theory, which uses the concept of differential calculus to estimate and conclude inductively.

Epistemologically, Statistics connects the real world with the theoretical world. An example is statistical modeling, where the statistician assumes a stochastic model and uses samples to estimate parameters. The model of understanding and thinking of student statistics promotes students' statistical thinking based on philosophical and historical statistics. Students' ways of understanding and Thinking about Statistics have implications for the didactic design of Statistics. The didactic design should minimize the gap between students' statistical thinking and philosophical inference inductive Statistics (Statistical Epistemology). The findings of this study can be considered in designing learning and in measuring students' statistical abilities.

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