





avoiding the combinatorial explosion in the number of free parameters that occurs in FLNN.

As shown in Fig. 3, PSNN consists of two layers; the product layer and the summing layer. The trainable weights are found only between the inputs and the summing units. The structure of PSNN is highly regular due the fact that the summing units can be added incrementally until a specified goal is achieved.

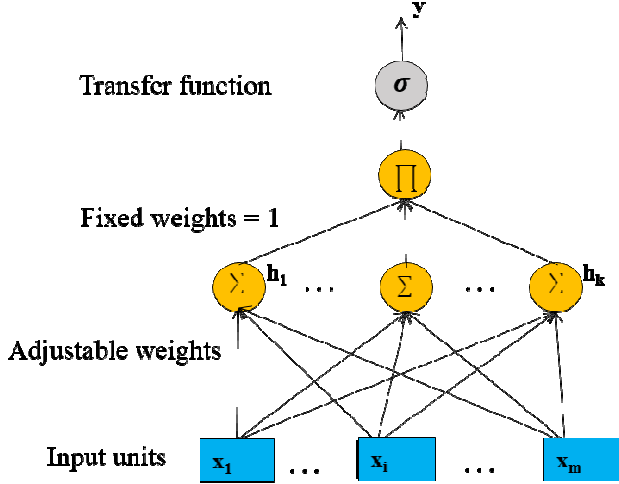


Fig. 3 Pi-Sigma neural network.

Despite the fact that PSNN is not a universal approximator [27], it demonstrated competent ability to deal with many problems such as classification [28], time series forecasting [19], image coding [29] and visual cryptography [30].

The learning algorithm for PSNN using the incremental backpropagation algorithm is as follows:

For a given input,

- Calculate the output as follows:

$$y = \sigma\left(\prod_{j=1}^k h_j\right) \quad (4)$$

$$h_j = \sum_{i=1}^m W_{ij} X_i + W_{0j} \quad (5)$$

where  $\sigma$  is an activation function,  $W_{0j}$  are the biases,  $W_{ij}$  are weights that link input nodes with the summing nodes,  $x$  is a component of input vector  $X$ .

- Compute the weight changes:

$$\Delta W_i = \eta(d_i - y_i) y_i (1 - y_i) \prod_{j \neq i}^M h_j X_k \quad (6)$$

where  $\eta$  is the learning rate and  $d$  is the desired output.

- Update the weights:

$$W_i = W_i + \Delta W_i \quad (7)$$

- Continue until termination condition is satisfied.

#### D. Experimental Design:

1) *Time Series benchmark data:* we used two benchmark chaotic time series, namely the monthly smoothed sunspot numbers and the Mackey-Glass time-delay differential equation time series.

Sunspot time series is a good indication of solar activity for solar cycles [31]. It is very important to forecast Sunspot time series due to the observed impact of solar activity on earth, climate, weather, satellites and space missions [31].

In this paper, we downloaded the monthly smoothed sunspot time series from [32]. To compare the performance of FLNN and PSNN with other models in [31], two thousands points from November 1834 to June 2001 were selected.

Mackey-Glass time series is a benchmark problem that has been used by many researchers [11]–[14]. This time series is given by the following delay differential equation:

$$\frac{dx}{dt} = \beta x(t) + \frac{\alpha x(t - \tau)}{1 + x^{10}(t - \tau)} \quad (8)$$

where  $\alpha=0.2$ ,  $\beta=-0.1$ ,  $x(0)=1.2$ , and  $\tau=17$ . With this setting the series produce chaotic behaviour and we can compare the forecasting performance of FLNN and PSNN with other models in the literature. This time series can be found in mgdata.dat in MATLAB [33].

The input-output data pairs and the number of training and testing samples that we used in this paper for these two time series are shown in Table I. Fig. 4 and Fig. 5 show the used interval for training and testing samples for both time series.

TABLE I  
TIME SERIES INFORMATION

Time series	Input-output data pairs	Training samples#	Out-of-sample Samples
Sunspot	$[x(t - 4), x(t - 3), x(t - 2), x(t - 1), x(t); x(t + 1)]$	1000	1000
Mackey-Glass	$[x(t - 18), x(t - 12), x(t - 6), x(t); x(t + 6)]$	500	500

2) *Data Preprocessing:* We scaled the points to the range [0.2, 0.8] because we used the sigmoid activation function. We used the minimum and maximum normalization method which is given by:

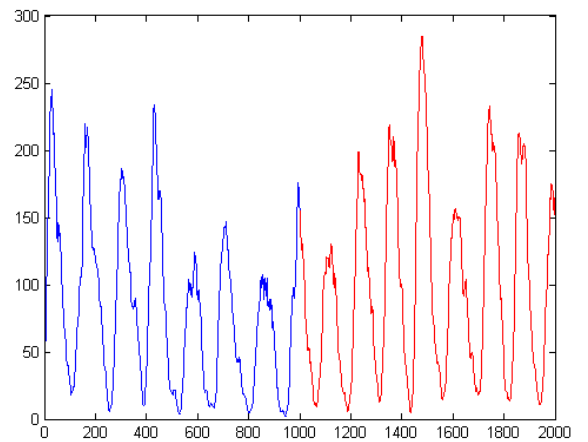


Fig. 4 Sunspot time series. Blue points for training while red points for testing.

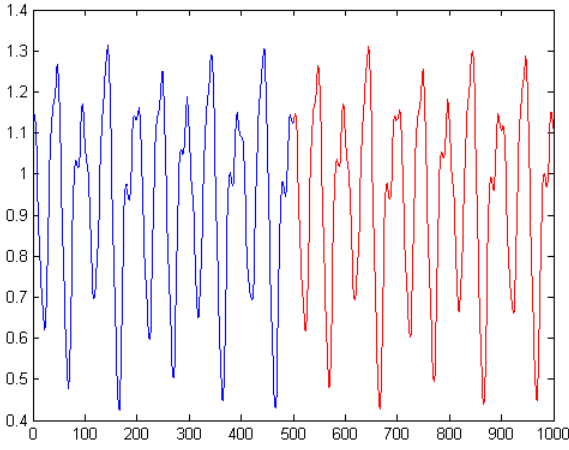


Fig. 5 Mackey-Glass time series. Blue points for training while red points for testing.

$$\hat{x} = (\max_2 - \min_2) * \left( \frac{x - \min_1}{\max_1 - \min_1} \right) + \min_2 \quad (9)$$

where  $\hat{x}$  is the normalized value of  $x$ ,  $\min_1$  and  $\max_1$  are the minimum and maximum values of all observations, and  $\min_2$  and  $\max_2$  refer to the minimum and maximum values of the new range.

3) *Network Topology and Training*: The topology of the FLNN and PSNN that we used is shown in Table 2. Most of the settings are selected empirically.

TABLE II  
NETWORK TOPOLOGY

Setting	Value
Activation function	Sigmoid function
PSNN order	Empirically selected from 2 to 15
FLNN order	Empirically selected between 2-5 for Sunspot and between 2-4 for Mackey-Glass
Stopping criteria	Maximum number of epochs =3000
Initial weights	[-0.5,0.5]
Learning rate	[0.01-1]
Momentum	[0.4-0.8]

4) *Performance Metrics*: Due we aim to compare the forecasting performance of FLNN and PSNN with other models in the literature, we used the Normalized Mean Squared Error (NMSE) and the Root Mean Squared Error (RMSE) metrics. NMSE and RMSE are given by:

$$NMSE = \frac{1}{N\sigma^2} \sum_{i=1}^N (y_i - \hat{y}_i)^2$$

$$\sigma^2 = \frac{1}{N-1} \sum_{i=1}^N (y_i - \bar{y})^2 \quad (10)$$

$$\bar{y} = \frac{1}{N-1} \sum_{i=1}^N y_i$$

$$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (y_i - \hat{y}_i)^2} \quad (11)$$

where  $N$ ,  $y$  and  $\hat{y}$  represent the number of out-of-sample data, actual output and network output, respectively.

### III. RESULTS AND DISCUSSION

The forecasting models for FLNN and PSNN of the two time series are built via the experimental design settings. In order to obtain fair comparison between FLNN and PSNN and avoid weight initialization influence, the average performance of 30 simulations are reported as shown in Table III and Table IV. Note that, the results that are shown in these two tables are the de-normalized results. That means, we de-normalized the forecasted value and compared it with the original desired value before calculating the used metrics. Best average results are in boldface.

As can be seen from Table III and Table IV, FLNN outperform PSNN on Sunspot time series, while PSNN is better than FLNN on Mackey-Glass. Therefore, each one has its ability based on the time series properties.

TABLE III  
AVERAGE RESULTS FOR SUNSPOT TIME SERIES

HONN model	Network order	Number of parameters	RMSE	NMSE
FLNN	5	32	<b>2.7398</b>	<b>0.0015</b>
PSNN	2	12	4.6745	0.0045

TABLE IV  
AVERAGE RESULTS FOR MACKEY-GLASS TIME SERIES

HONN model	Network order	Number of parameters	RMSE	NMSE
FLNN	3	15	0.0369	0.0267
PSNN	6	30	<b>0.0129</b>	<b>0.0033</b>

During the simulations, we noticed that increasing network order of PSNN results in decreasing forecasting performance on Sunspot time series but it helps PSNN on Mackey-Glass time series. Learning curves for the best simulations are shown in Fig. 6 and Fig. 7. It can be seen that there is no much improvement in learning after 500 epochs. Note that, we do not de-normalize the RMSE in Fig. 6 and Fig. 7.

The best performance with FLNN and PSNN using the out-of-sample data for Sunspot and Mackey-Glass time series are shown in Fig. 8 to Fig. 11. As it can be noticed from these figures that FLNN and PSNN to some extent can follow the dynamics behaviour of the time series.

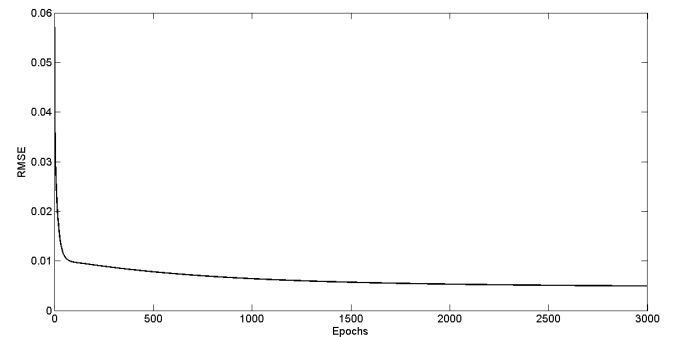


Fig. 6 Learning curve for best FLNN simulation on Sunspot time series.

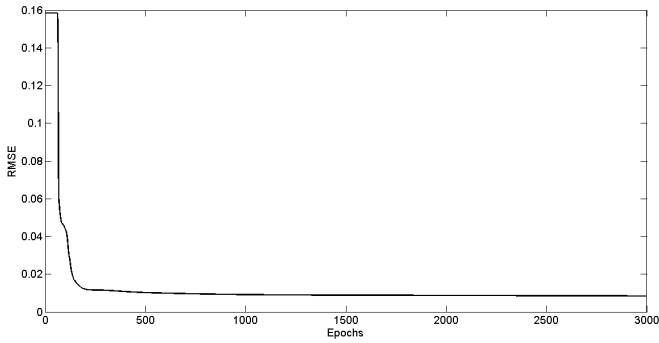


Fig. 7 Learning curve for best PSNN simulation on Mackey-Glass time series.

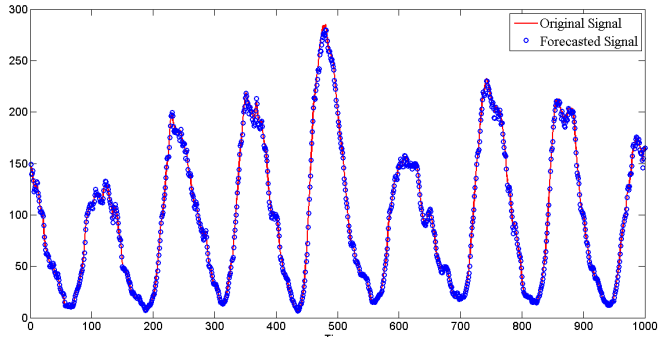


Fig. 8 Out-of-sample forecasting for best FLNN simulation on Sunspot time series.

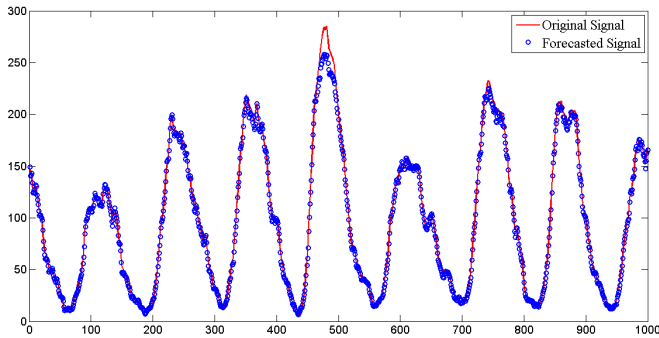


Fig. 9 Out-of-sample forecasting for best PSNN simulation on Sunspot time series.

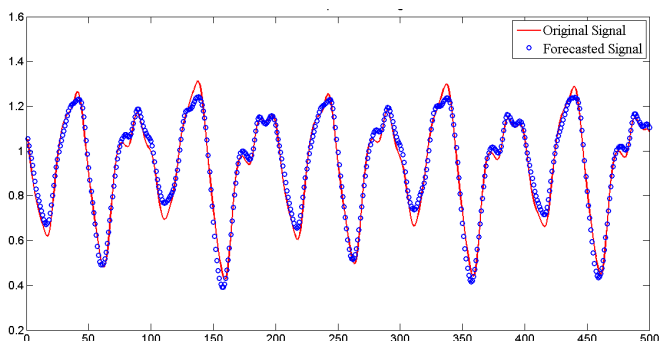


Fig. 10 Out-of-sample forecasting for best FLNN simulation on Mackey-Glass time series.

Finally, a comparison among different models in the literature with FLNN and PSNN is shown in Table V and Table VI. It should be noted that based on our search we could not find studies that used the normalization range that we used in this work. For that, we used the de-normalized results for best FLNN and PSNN simulations and compared them with the de-normalized results in the literature or with

studies that did not use any normalization method. The results show that FLNN and PSNN offer good performance compared to other hybrid models in the literature. Therefore, hybridizing other models with FLNN or PSNN could enhance the forecasting performance.

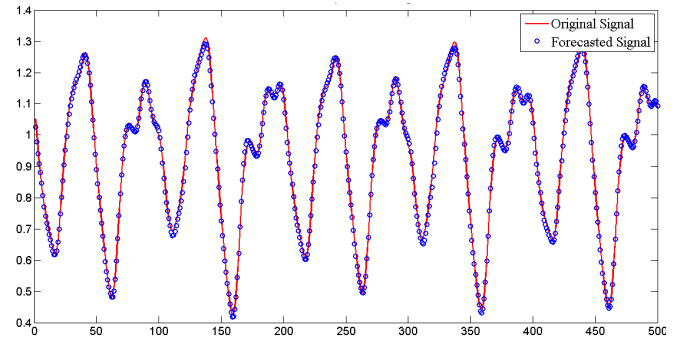


Fig. 11 Out-of-sample forecasting for best PSNN simulation on Mackey-Glass time series.

TABLE V  
COMPARISON OF THE PERFORMANCE OF VARIOUS EXISTING MODELS ON SUNSPOT TIME SERIES

Model	NMSE
Fuzzy neural networks (FNN) [31]	0.0174
AdaBoost.regression and threshold-FNN [31]	0.0160
Modified-AdaBoost.regression and threshold-FNN [31]	0.0135
<b>PSNN -Order 2 (this work)</b>	<b>0.0044</b>
<b>FLNN -Order 5 (this work)</b>	<b>0.0015</b>

TABLE VI  
COMPARISON OF THE PERFORMANCE OF VARIOUS EXISTING MODELS ON MACKEY-GLOSS TIME SERIES

Model	RMSE
Fuzzy modelling method with Singular Value Decomposition (SVD) [7]	0.0894
Gustafson-Kessel fuzzy clustering method + Kalman Filtering Algorithm (KFA) with SVD [8]	0.0748
Orthogonal function neural network + recursive KFA based on SVD [34]	0.05099
Adaptive fuzzy inference system with local search for learning algorithm [9]	0.045465
<b>FLNN -Order 4 (this work)</b>	<b>0.03656</b>
Beta basis function neural networks + Differential evolution algorithm [11]	0.030
Dynamic evolving computation system [35]	0.0289
Backpropagation network + hybrid K-means-greedy [14]	0.015
Modified differential evolution + radial basis function [13]	0.013
<b>PSNN -Order 6 (this work)</b>	<b>0.0118</b>
Takagi-Sugeno fuzzy system-singleton + simulated annealing [10]	0.00898

#### IV. CONCLUSIONS AND FUTURE WORKS

This paper presents the application of two higher order neural networks, namely functional link neural network (FLNN) and pi-sigma neural network (PSNN) to forecast two benchmarks chaotic time series; the monthly smoothed sunspot numbers and the Mackey-Glass time series. Results showed that FLNN outperforms PSNN on Sunspot time

series while PSNN is better than FLNN on Mackey-Glass time series. Furthermore, a comparison with different hybrid models in the literature showed that FLNN and PSNN offer good performance compared to these hybrid models. Future works could be hybridizing swarm intelligence techniques with FLNN or PSNN and applying them to forecast chaotic time series.

#### ACKNOWLEDGMENT

The authors would like to thank Universiti Tun Hussein Onn Malaysia (UTHM) and Ministry of Higher Education (MOHE) Malaysia for financially supporting this research under the Fundamental Research Grant Scheme (FRGS), Vote No. 1235.

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