

## New Constants of Fracture Angle on Quartz Sandstone

Supandi<sup>a,1</sup>, Zufaldi Zakaria<sup>b</sup>, Emi Sukiyah<sup>c,1</sup>, Adjat Sudradjat<sup>c,2</sup>

<sup>a</sup>Department of Mining Engineering, Faculty of Technology Mineral, Institut Teknologi Nasional, Yogyakarta, Indonesia  
E-mail: supandi@itny.ac.id

<sup>b</sup>Department of Applied Geology, Faculty of Geological Engineering, Padjadjaran University, Bandung, Indonesia  
E-mail: zufaldi.zakaria@unpad.ac.id

<sup>c</sup>Department of Geoscience, Faculty of Geological Engineering, Padjadjaran University, Bandung, Indonesia  
E-mail: <sup>1</sup>emi.sukiyah@unpad.ac.id; <sup>2</sup>asudradjat@yahoo.com

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**Abstract**— Calculation of fracture angle based on the Mohr-Coulomb concept has been generalized to all lithologies, whereas each lithology has a heterogeneous characteristic that is not the same as one another. This research aims to determine constants that calculating the fracture angle based on the Mohr-Coulomb concept for quartz sandstone. The fracture angle based on the results of calculations using mathematical formulas and measurement results in the laboratory is compared. The empirical result for analysis was obtained from direct measurement under the uniaxial test with a single fracture and compared them based on mathematical calculation. Laboratory testing was applied to 50 mm-sized core samples of sandstone. The value of the fracture angle was obtained after taking uniaxial testing with fracture angle measurement against the plane directly. The determination of the friction angle was performed using an undrained-unconsolidated triaxial method. The determination of the correlation constants was conducted by plotting the scatter graph between the friction angle and fracture angle. The correlation shows that the fracture angle has a very strong relationship with rock characteristics. The constants based on the laboratory test are higher than that of the Mohr-Coulomb concept. The increasing constants depend on several factors: mineralogy, cohesion, internal friction angle, density, void ratio, and Poisson's ratio. The constants of the fracture angle for the quartz sandstone should be changed from 45 to 53.705. This result may be proof that generalizing the formula of fracture angle to all lithologies may not be applied due to the differences of their characteristics in defining the fracture angle.

**Keywords**— friction angle; fracture angle; Mohr-Coulomb; uniaxial test; quartz sandstone.

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### I. INTRODUCTION

Rock engineering properties are considered to be the most critical parameters of the mechanical properties of rock, and failure criterion is mainly obtained from triaxial laboratory testing. The most widely used failure criterion is the Mohr-Coulomb criterion. To obtain the Mohr-Coulomb failure envelope, a conventional triaxial test is used. Conventional triaxial testing is simple but requires multiple samples. Aside from limited availability, multiple samples also provide potential uncertainty in the resulting parameters due to sample heterogeneity, as different samples might have significant variations in strength. [1] carried out an analysis of the deformation characteristics of bedded sandstone. They determined the deformation of sandstone by using an optical deformation and strain measuring system during uniaxial testing. Increasing stress occurred at the bedding contact, but the fracture angle has not been determined yet. The determination of the fracture angle of massive sandstone has

not been tested yet. The peak strength of porous sandstone depends on porosity, pore angle, young modulus, and pore ratio [2].

Deformation under uniaxial and Triaxial depends on some factors such as isotropic material, mineralogy, strain, passion ratio, and fissure [3]. On the homogenous sandstone strains up to 0.2%, but this number will increase as long as the clay content increases. The determination of sandstone material based on size distribution and mineralogy is ignored. Failure model of the specimen depends on the type of rock, mineralogy of rock, and stress condition [4]. Existing fissure and lamination also contribute toward failure mode. Comparison between hyperbolic failure and establish failure criteria for cohesive –frictional material. The study is carried out through different lithology and one cement but not detailing to measure fracture angle [5]. Calibrate of fracture angle is required for each material since it has a heterogeneous characteristic, so engineering design will be resulting in realistic results refers to each lithology. The sliding body of the slide plane is dependent on some

contribution factors; one of them is crack shape. Crack shape involves position, angle of cracking, and shape roughness. The increasing angle of rock crack depends on the crack shape on the reliability index [6]. The failure criterion is formulated by comparing the available energy release rate due to the presence of a crack in the matrix and fracture toughness in the interface. The parameter of the failure criterion is dependent on material properties, ply thickness, and laminate compliance properties before and after the presence of a matrix crack [7].

The calculation of the fracture angle has not been measured yet. Peak strength and crack damage of sandstone is depending confining pressure, and it can be described by non-linear Hoek-Brown Criterion than by linear Mohr-Coulomb Criterion [8]. Cohesion and internal friction angle decreased with the increasing exposure time [9], [10]. Decreasing cohesion significantly is occurred after 100days after exposing, and it will be decreasing as long as exposure time. According to the classification by Dickinson and Suczek, the provenance of Warukin sandstones is generally recycled orogen with subclassification of quartzose recycled [11].

The maximum fracture angle failure criterion for shale was about  $60^\circ$  [12]. The number could not be generated for other material since some other material showing a lower number such as gneiss is showing a fracture angle about  $45^\circ$ . The number of fracture angle depends on the deformability of rock, modulus, strength, permeability, and orientation of plane [13], [14]. Modified failure criterion shall be conducted on the isotropic rock for different plane bedding shares. Correction number shall be applied while testing conducted on different bedding shares [15]. Comparison failure criterion is carried out toward fifteen failure criteria and resulting not better failure criterion for any materials. Each lithology having a unique failure criterion and development failure criterion for each lithology can apply [16].

The new failure criterion has been studied by many researchers, i.e. [17]–[25]. The fracture angle depends on the degree of saturation, changing strain, and water saturation, and it occurred special at the soft rock [26]. The overall stress-strain behaviors of rocks are then obtained using the crack damage volume fraction. The theoretical results are found to be in good agreement with experimental data [27]. Density is having a contribution toward the changing fracture angle on concrete [28].

The micromechanical grain of the hard rock should consider:

- Grain size distribution
- Mineralogy of rock
- Plasticity
- Creeping or strain
- Crack growing
- Intra granular fracturing
- Shear and tensile cracking
- Macroscopic wing rock
- Primary and secondary creeping

This study relies on the previous study on microstructure [29], [30]. Mineralogy is the main factor toward the behavior of granite, and it will be affected toward strength, elastic, and microstructure [31]. Confining pressure and hole

diameter will be affected by Poisson's ratio and peak strain of hollow sandstone [32]. Stress-strain of soft rock with porous material is having a correlation with micromechanics based on the thermodynamics damage model [27]

Pore geometry effects have important implications for rock strength in general, in addition to the maintenance of open pore space, which in turn contributes to the long-term maintenance of permeability in the subsurface [33]. The friction angle will be decomposed into a critical stage friction angle and a portion of the dilatancy angle to capture the peak phenomenon of dilative sand and resulting in better performance than the conventional Mohr-Coulomb Model [24].

The uniaxial strength of rock with one set of joints is related to the inclination and number of joints. As the number of joints increases, the rock strength gradually decreases and becomes approximately constant. The fracture angle has not been determined yet. Loading conditions under the uniaxial test is having a significant impact on crack patterns and shear crack [34]. Three types of characteristic stresses present an increasing flaw angle. Increasing the flaw angle will be responded by increasing stress [35].

Opening rock under the uniaxial test is having a significant effect on rock mechanical behavior. Decreasing rock strength will be followed by decreasing the young modulus [36]. Increasing the elasticity modulus and Poisson's ratio will be followed by increasing confining pressure. The relationships between the friction angle and the cohesion, strength, and bedding plane angle are the same as that with the elasticity modulus. The strength reduction factor is decreasing as long as the increase of joint orientation angle (degree), and it is important to pay attention to the determination fracture angle of rock slide [37].

The angle of internal friction intended to measure the ability of a unit of rock or soil to withstand shear stress. It is the angle ( $\phi$ ), measured between the normal force (N) and resultant force (R), that is attained when failure just occurs in response to a shearing stress (S). Its tangent (S/N) is the coefficient of sliding friction. Its value is determined experimentally. Geopolymerization is giving significant impact on the increasing shear strength under the Unconfined Compression strength test for stabilized fine-grained soil [38]. Surface roughness gives a significant effect on shear strength and failure criterion of claystone. The rough surface is resulting in higher cohesion and friction angle compares to a smooth surface [39]. Fracture angle in nature has an important effect on rock behavior. The fracture angle of anisotropic rock is related to the bedding plane. The failure pattern of uniaxial compression is generally tensile failure or tensile failure followed by shear failure. The fracture angle decreases with the increase in confining pressure [40].

## II. MATERIAL AND METHOD

New constants predict the fracture angle of quartz sandstone based on the numerical method proposed by Mohr-Coulomb. The existing parameter to calculate the fracture angle is based on the Mohr-Coulomb method using  $45^\circ$  without mention on the specific rock. The numerical method based on Mohr-Coulomb methods is not applicable

for all materials because each material has special fracture angle characteristics that depend on several factors. Quartz sandstone also has a special characteristic related fracture angle under laboratory test.

The sandstone materials used in this study were collected from the Kusan Block at Tanah Bumbu, South Kalimantan, Indonesia. The samples were collected from a part of the mine slope approximately 50 cm length; undisturbed samples were collected with 70 mm diameters. The samples were wrapped using plastic to maintain their basic properties. The rock samples used in this study were 40 samples of sandstone with medium-fine grain sizes. Sample preparation was conducted by conforming to the ASTM (American Society for Testing and Materials)-specified ASTM standards D4220-95.

The samples were grouped based on visual descriptions carried out to determine the homogeneity of the samples using laboratory testing. The focus of this study was on the behavior of quartz sandstones in the Warukin formation located in the Kusan Blok. XRD analysis was required to determine the mineral content of the rock. Based on the XRD analysis, quartz is the predominant rock component, and the quartz content exceeds 90% (Fig. 1). Several analyses of the sandstone resulted in a similar composition, and the sandstone mineralogy is likely homogenous, so the assessment of the mechanical properties will be the next focus. The mineral composition of the sandstone is shown in Fig. 2.



Fig. 2 Quartz grains overview under a microscope

The fracture angle of the rock was obtained using uniaxial compression testing, referring to ASTM D4405-93 on uniaxial compression testing of the elastic modulus for rock core specimens. The test used devices with an axial loading range of 0-4 600 kN (compression) and axial displacement range of 0-100 mm ( $\pm 50$  mm). The sensor accuracy in each test was approximately 0.5%, and the traversal deformation was approximate -2.5-12.5 mm. Uniaxial testing will obtain the fracture angle at peak strength. Determination of the type of fracture angle has been described in the previous study [41]. The fracture angle can be divided into four types:

- Single fracture
- Parallel fractures
- Intersecting fractures
- Mixed fractures

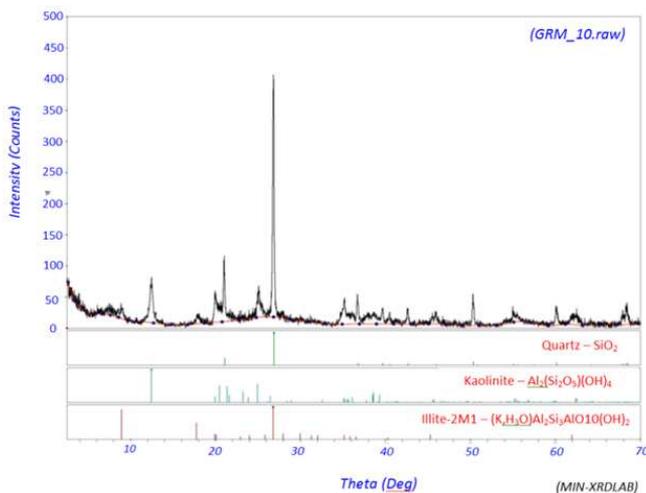


Fig. 1 XRD Spectrograph, it appears that quartz minerals dominate sandstones.

The rupture angle and shear angle were assessed using triaxial testing. The results from the triaxial tests were used to obtain the cohesion and friction angle. Triaxial compression shear tests (UU) were performed when the samples were unconsolidated and undrained to determine the friction angle ( $\phi$ ) and cohesion (c) under maximum stress conditions, and this method is effective because it is approximately the field conditions. The research was conducted by laboratory analysis with an empirical triaxial compression test method, referring to ASTM D.2850-87.

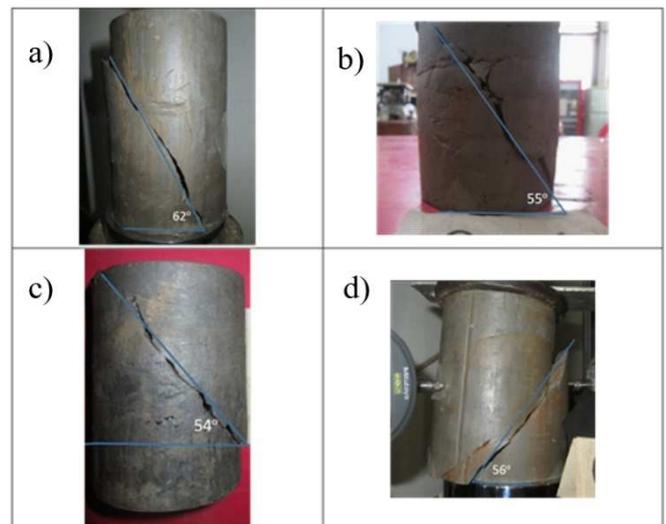


Fig. 3 Measurement fracture angle on quartz sandstone after uniaxial test resulting in different fracture angles. (a) fracture angle 62°, (b) fracture angle 55°, (c) fracture angle 54°, and (d) fracture angle about 56°.

This research was focused on measuring the fracture angle of a single fracture type. The fracture angle magnitude was obtained by manually measuring the crack zone from horizontal for all the samples tested. Figure 3 shows

examples of the measurement result. Measurements were collected from all existing sandstone samples in order to perform an empirical analysis.

Based on the achievements of earlier researchers, this paper will continue to investigate the constants controlling the fracture angle under uniaxial testing. This study proposes to ensure that the constants used to calculate the fracture angle from the Mohr-Coulomb concept is not applicable on any materials. Each material is suggested to have a specific constants to determine its fracture angle. Additionally, this research investigates factors that will affect this constants. The experimental results will provide a reference for engineering practice, especially for slope stability.

### III. RESULTS AND DISCUSSION

Mineralogical testing obtained a quartz composition above 90%; therefore, the mineralogy of the sandstone is homogeneous. A statistical test was conducted to ensure that the tests were conducted on the same rock group. A test was carried out on the quartz mineral composition. The first statistic test was a rock normality test.

The normality and homogeneity test was carried out determination characteristic of quartz sandstone based on mineral composition. This test is important to make sure that the analysis is carried out on similar mineralogy and sample having similar characteristics. The normality test is resulting in quartz sandstone having normal distribution based on quartz content. The homogeneity test is also resulting in quartz sandstone from the Kusan block is homogenous or uniform.

Based on the statistical tests showing the same characteristics, the fracture angle magnitude is controlled by the physical rock characteristics. To determine the constants that affect the fracture angle, plotting of all the constants tested was performed. A strong correlation degree is considered to indicate a factor affecting the difference between mathematical and empirical values.

In the Mohr-Coulomb concept, the greatest stress is  $\sigma_1$ , while  $\sigma_3$  is the smallest stress. When  $\sigma_1$  is applied to a sample and is greater than the strength of the rock, a fault will form. Mathematically, Mohr-Coulomb considers that the fracture angle will be formatted at a  $45^\circ$  angle from half of the friction angle. However, this value is highly dependent on the rock properties, so that in this study, it will be empirically assessed whether sandstone has a fracture angle that follows the Mohr-Coulomb theory or other equation.

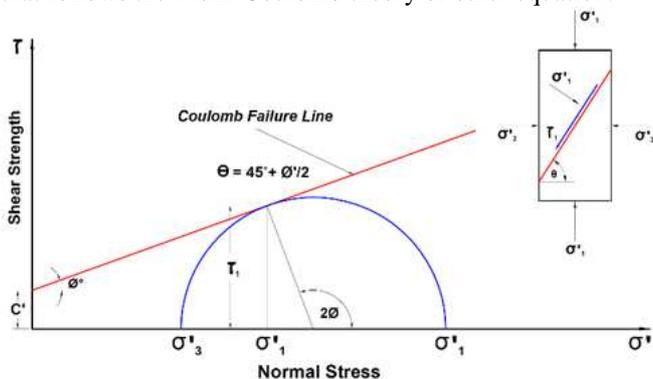


Fig. 4 Mohr-Coulomb concept of failure angle [42]

The Mohr-Coulomb concept illustrated in Fig. 4 explains that  $\phi$  is the fracture angle obtained from the horizontal. The fracture angle is a function of the combination of rock strength and main source emphasis. Cohesion is determined by the line that intersects the shear strength along the Y-axis.

Based on this study, the friction angle is correlated with the cohesion, hardness, and another physical rock characteristic.

In the first stage, the fracture angle is calculated based on the friction angle using the Mohr-Coulomb formula. The fracture angle is determined by:

$$\text{Fracture Angle} = 45^\circ + \phi/2 \quad (1)$$

Plotting in a scatter diagram results the following equation:

$$y = 0.5x + 45 \quad (2)$$

The second analysis was conducted by plotting the friction based on undrained-unconsolidated triaxial testing with the fracture angle based on uniaxial testing. Measurement of the fracture angle was conducted manually based on the fracture shape after the uniaxial tests. Measurements were conducted on the 38 samples tested. The measurement results provide the fracture angle for each friction angle (Fig. 5).

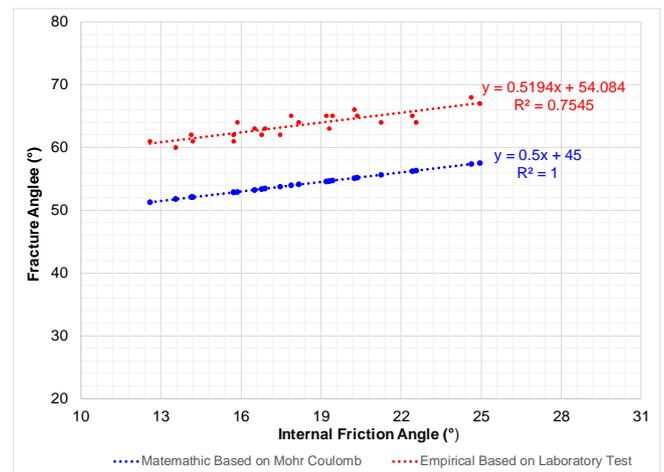


Fig. 5 Comparison of the fracture angle between the Mohr-Coulomb concept and empirical analysis based on laboratory test

The degree of correlation was approximately 0.75 and categorized as a strong correlation. The equation based on the correlation is:

$$y = 0.5215x + 53.705 \quad (3)$$

These two correlations generate parallel lines and the same type of relationship. The difference between the equations is constant. The fracture angle based on the Mohr-Coulomb concept is approximately  $45^\circ$ , while the result of the fracture angle based on the laboratory data is approximately  $53.705^\circ$ .

Furthermore, the influence of several factors with possible constant changes in value was analyzed, and some limitations of the constants were generated from the empirical measurements. The analysis was performed by a correlation between the fracture angle and several

mechanical and physical characteristics. The correlation analysis was performed by plotting the fracture angle with several factors.

The first analysis was plotting the fracture angle with the rock strength. The plotting result is shown in Fig. 6. The results from this test were plotted in a diagram to determine the correlation between the fracture angle and rock strength. The degree of correlation was 0.8411 (robust correlation) with the regression correlation fracture angle and strength expressed;

$$y = 5E - 23x^{13.7748} \quad (4)$$

Fig. 6 shows that the fracture angle increased significantly with strengths between 30 and 50 kPa and that this trend decreased with strengths up to 50 kPa.

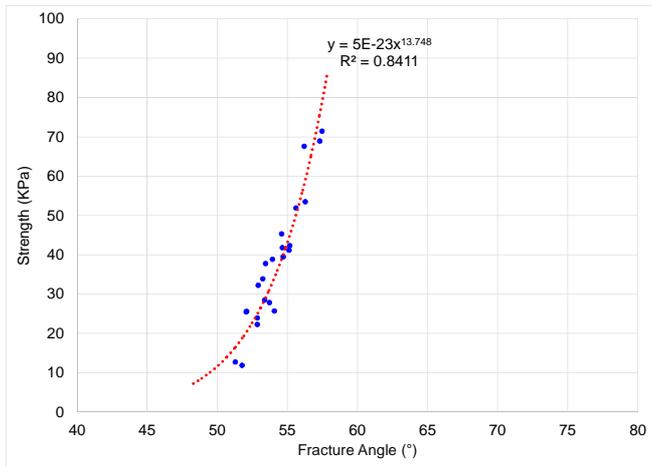


Fig. 6 Relationship of fracture angle with strength

The other analysis was the correlation with cohesion. The plotting result is shown in Fig. 7. The degree of correlation shows a strong correlation in the amount of 0.6702. Figure 7 also indicates that the cohesion also increased from 50 to 250 kPa, and the degree of correlation becomes strong. Based on this study, the cohesion has a role in changing the fracture angle.

The fracture angle is correlated to the physical rock characteristics, and the contribution of each factor is obtained. The first fundamental property correlated with the fracture angle is the moisture content. The moisture content is plotting on the vertical axis, as shown in Fig. 8. It is shown that the moisture content plays a considerable role in changing the fracture angle by a constant value. The correlation value between these two parameters was 0.777, meaning that there is a strong correlation. When the moisture content decreases, the fracture angle increases. Decreasing moisture content simultaneously increases the rock strength; therefore, it is concluded that the moisture content has a strong influence on the fracture angle. The correlation fracture angle with moisture content expressed:

$$y = 291753x^{-2.379} \quad (5)$$

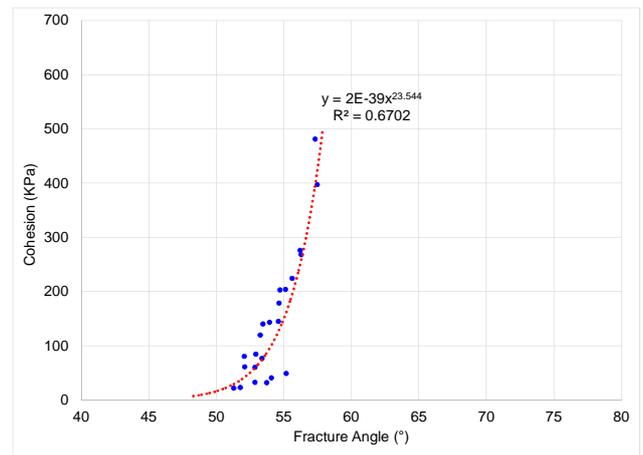


Fig. 7 Relationship of fracture angle with cohesion

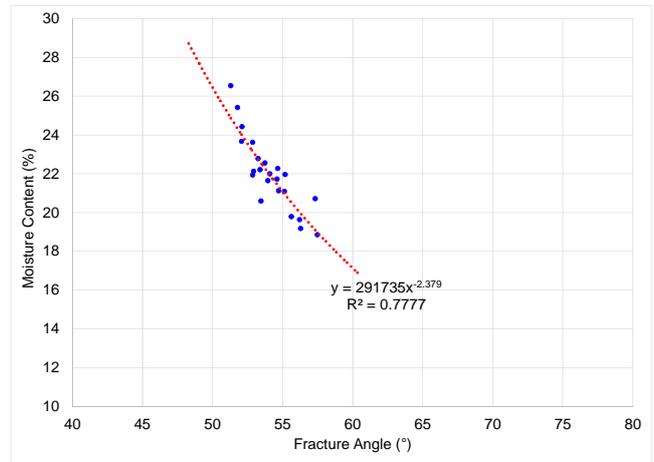


Fig. 8 Relationship of fracture angle with moisture content

A correlation is also conducted for the fracture angle and void ratio, and the graph below shows a degree of correlation of 0.6813, which indicates a strong correlation. It was observed that when the void ratio decreases, the fracture angle increases (Fig. 9). The correlation equation is as follows:

$$\text{Void Ratio} = 9060x^{2.411} \quad (6)$$

Another factor is Poisson's ratio, which is the ratio of the fracture angle axial strain and diametric strain. This value needs to be analyzed because it theoretically has a strong correlation to the stress distribution. When rocks are forced before fracturing, they will respond to the strain change. Based on this condition, it is important to know the fracture angle correlation between Poisson's ratio and the fracture angle.

The result of the correlation is shown in Fig. 10, where the correlation between the fracture angle and Poisson's ratio denotes a moderate relationship with a degree of correlation of 0.4385. The greater the value of Poisson's ratio is, the greater the generated fracture angle will be. From the correlation output the correlation formula is obtained.

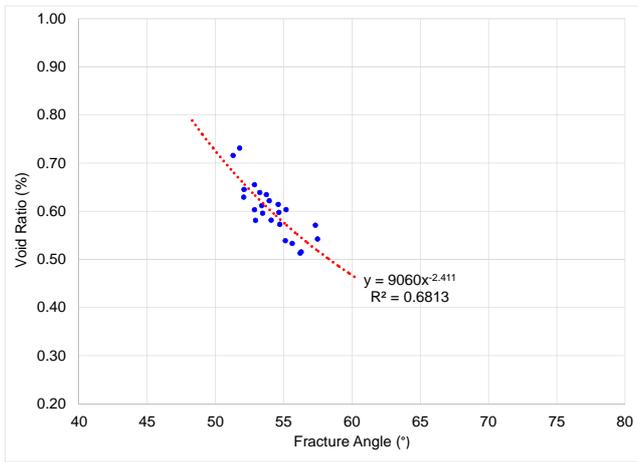


Fig. 9 Relationship of fracture angle with void ratio

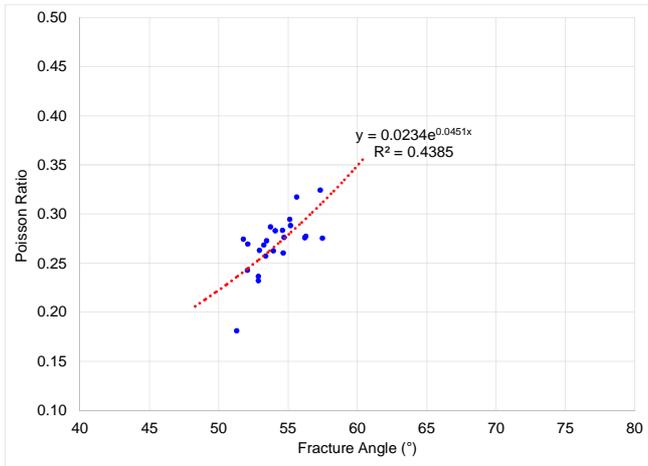


Fig. 10 Correlation of fracture angle with Poisson's ratio

Other physical inputs into this analysis are the wet and dry density. The fracture angle is predicted to be associated with the rock mass density; therefore, the correlation between the fracture angle and density needs to be understood (Fig. 11). Increasing wet and dry density will be followed by increasing fracture angle with function is relatively linear. Fracture angle about 53-58° has occurred at the wet density of around 1.8 – 2.2 gr/cm<sup>3</sup>. Wet density and dry density are having strong relationships with moisture content and refer to Fig. 8 that moisture content having a strong relation with fracture angle, so density has a strong relation with fracture angle is reasonable.

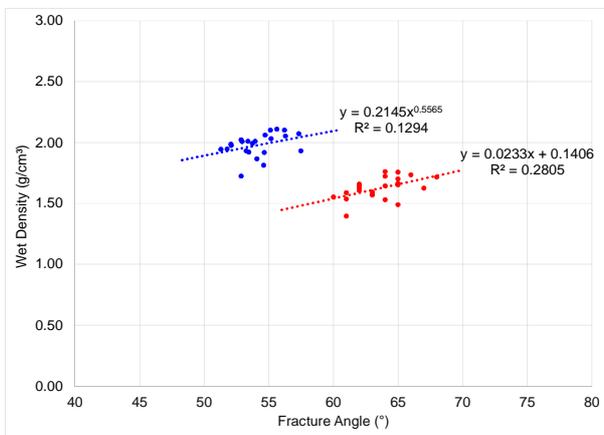


Fig. 11 Correlation of fracture angle with wet/dry density

The degrees of correlation for the wet and dry densities were 0.1294 and 0.2805, respectively. From this degree of correlation, a poor relationship between fracture angle and rock density was obtained. The correlation equations for the wet and dry densities are:

$$\text{Wet Density} = 0.2145x^{0.5655} \quad (7)$$

$$\text{Dry Density} = 0.233x^{1.406} \quad (8)$$

The fracture angle of the quartz sandstone did not depend on the axial strain since the degree of correlation is very small (0.0007). Fig. 12 shows the distribution of axial strain, which does not seem to have a good relationship with the fracture angle. Rock strain did not have a correlation with axial strain but fracture angle having good relation with mechanical properties such as cohesion, internal friction angle, and strength.

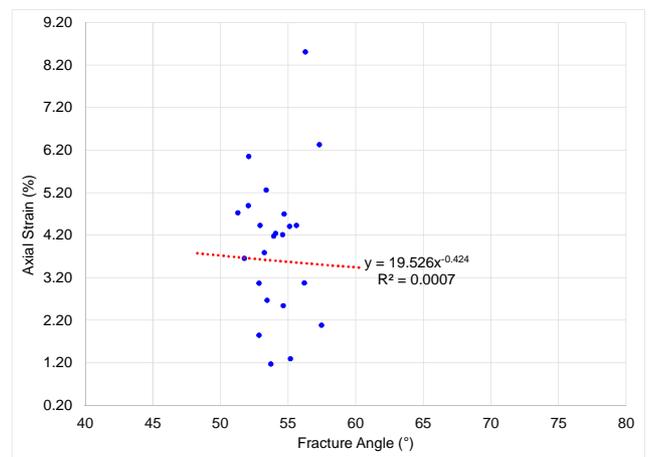


Fig. 12 Relationship of fracture angle with axial strain

#### IV. CONCLUSIONS

The fracture angle of sandstone is higher than that calculated with the Mohr-Coulomb equation  $y = 0.5x + 45$  with  $r^2 = 1$ . However, the research test results of 38 samples of sandstone from the Warukin formation showed that the fracture angle is described by  $y = 0.5215x + 53.705$  with  $r^2 = 0.75$ . The test result of the constants (53.705) is greater than the calculated Mohr-Coulomb constants (45). The differences are affected by the following factors: strength; Cohesion; Void Ratio; Poisson's ratio; density.

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