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Gravitational Base Parameters Identification for a Knee Rehabilitation Parallel Robot

Pulloquinga José L. [a,*], Escarabajal Rafael J. [a], Mata Vicente [b], Valera Ángel [a], Zambrano Iván [c], Rosales Andres [d]

[a] *Departamento de Ingeniería de Sistemas y Automática, Universitat Politècnica de València, Camino de Vera, s/n, Valencia, 46022, Spain*
[b] *Departamento de Ingeniería Mecánica y de Materiales, Universitat Politècnica de València, Camino de Vera, s/n, Valencia, 46022, Spain*
[c] *Departamento de Ingeniería Mecánica, Escuela Politécnica Nacional, Ladrón de Guevara, 253, Quito, 170517, Ecuador*
[d] *Departamento de Automatización y Control Industrial, Escuela Politécnica Nacional, Ladrón de Guevara, 253, Quito, 170517, Ecuador*

*Corresponding author: *jopulza@doctor.upv.es*

***Abstract*— New robotic technology is emerging nowadays to tackle lower limb rehabilitation issues. However, the commercial robots available for lower limb rehabilitation are usually oversize and expensive. Knee rehabilitation is generally aided by a professional therapist, making this clinical procedure an interesting scope for robotics. Parallel robots are suitable candidates for knee rehabilitation due to their high load capacity, stiffness, and accuracy compared to serial ones. In contrast, this robot has singular configurations inside its workspace, and its dynamic model is generally complex. For these reasons, a parallel robot for knee rehabilitation needs an advanced control unit to solve complex mathematical problems that ensure patient security. This study proposes the base parameters identification of a compact gravitational linear model of a 3UPS+RPU parallel robot using singular value decomposition. This paper recommends adding a statistical method focused on condition number minimization to the singular value decomposition process. This statistical method reduces the computational resources taken searching for the best inertial parameters combination at the beginning of the base parameter identification. The gravitational base parameters identified have a physical meaning and low complexity. This fact makes the results of this research the basis of an adaptative control applied to 3UPS+RPU parallel robot. This study shows that the gravitational term is the most influential for knee rehabilitation tasks, compared with the inertial, Coriolis, and centrifugal components, regarding the dynamic behavior of the parallel robot.**

***Keywords*— Base parameter; identification; parallel robot; knee rehabilitation; model-based control.**

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I. INTRODUCTION

Robots have recently supported clinical rehabilitation to improve the treatment and accelerate the recovery of patient injuries, reducing the labor-intensive operations of the therapist [1]. In lower limb rehabilitation, there are several robotic systems grouped into five main groups: *a)* treadmill gait trainers, *b)* foot-plate-based gait trainers, *c)* overground gait trainers, *d)* stationary gait trainers, and *e)* lower limb rehabilitation systems [2]. The groups *a-d* have a big volume, so they need to be stationary and represent an expensive cost. In group *e*, which includes knee rehabilitation, compact and portable equipment has been developed based on parallel robots (PRs).

PRs are composed of a mobile platform connected to a fixed and two or more kinematic chains [3]. In contrast to a serial robot, PR is a suitable candidate for ankle and knee rehabilitation because of its high load capacity, high accuracy, and low power consumption. Ankle rehabilitation requires two rotational movements and one translational movement. Then, the PRs used in this application needs at least three degrees of freedom (DOF). The 3-DOF PRs for ankle rehabilitation have been studied in previous studies [4]–[6]. On the other hand, knee rehabilitation requires two rotational and two translational movements, so 4-DOF PRs are needed [7]. A compact reconfigurable 4-DOF has been designed at the Universitat Politècnica de València for knee rehabilitation and diagnosis [8].

Since PRs for knee rehabilitation and diagnosis work with a human limb, the control system must be robust in time-variant parameters. An adaptive controller identifies these parameters using the dynamic model of the PR rewritten linearly concerning the inertial parameters [9]. However, the trajectories required to identify the time-variant parameters correctly are difficult to find, and not all the identified parameters are relevant [10]–[12]. Gautier [13] uses Singular Value Decomposition (SVD) to identify base inertial parameters. Díaz-Rodríguez *et al.* [10] developed an offline identification of the relevant parameters of a 3-DOF PR with a physical feasibility criterion and based on the SVD process proposed [13]. This process was successfully applied in an adaptative controller for ankle rehabilitation [14]. Lee and Patk [15] applied the Riemannian metric to improve the inertial identification process. Calderon and Piedrahita [16] studied the advantages and drawbacks of numerical and symbolic base parameter identification.

The SVD process developed [13] defines the base inertial parameters using a non-unique regression matrix. Díaz-Rodríguez *et al.* [10] determined every possible regression matrix that produces different base inertial parameters, and then with a physical feasibility criterion, selected the best ones. The necessity of determining all possible regression matrices entails a high computational time. This research proposes an improved regression matrix searching for the SVD identification procedure. The regression matrix searching improvement is achieved by dividing the possible combinations into subgroups and analyzing the condition number produced. In each algorithm iteration, the element that causes the resulting regression matrix to have the highest condition number is discarded. The discarded element is not considered for the next iteration, reducing the number of possible regression matrices and thus decreasing the computational cost. The proposed base parameters identification is applied in both simulated and real 4-DOF PR for knee rehabilitation and diagnosis.

This work is organized as follows: “II. Materials and Methods” introduces the dynamic model of the 4-DOF PR for knee rehabilitation and diagnosis, the SVD identification process with the new criteria, and the guideline employed to perform the base parameters identification. “III. Results and Discussion” show by simulation the principal influence of the gravitational term in the dynamic behavior of the 4-DOF PR under study. The gravitational base parameters identified are presented, these base parameters are identified by simulation and experimentation on the 4-DOF PR. Finally, “IV. Conclusion” presents the main conclusions of this research.

II. Materials and Methods

A. A dynamic model of the 4-DOF PR

Knee rehabilitation is based on performing fundamental knee movements, e.g., hip flexion, knee flexion-extension, and knee rotation [17], [18]. At the Universitat Politècnica de València, a 3UPS+RPU PR has been designed and built for knee rehabilitation and diagnosis [19]. The 3UPS+RPU PR has 4-DOF: two translational movements, one rotational movement about the Tibiofemoral plane, and a rotation about the Coronal plane (see Fig. 1). The letters U, S and R stand for universal, spherical, and revolute joint, respectively. The letter P represents the prismatic joint and there are underlined to indicates the actuated joints. The parameters that define the architecture of the PR are presented in Table I.

The location of each joint is represented by eleven q_{ij} four generalized coordinates represent general coordinates and the mobile platform location (x_m, z_m, θ, ψ), see Fig. 1. The actuated or independent generalized coordinates are grouped in $\vec{q}_{ind}$, whose size is $Fx1$. For the PR under study $F = 4$.

The dynamic model of the 3UPS+RPU PR using the Principle of Virtual Power and applying the D’Alembert’s Principle is defined as:

$$\vec{F}_{in} + \vec{F}_{cyc} + \vec{G} + \vec{F}_{ent} + \vec{F}_f = \vec{\tau} \qquad (1)$$

$\vec{F}_{in}$ are the inertial forces, $\vec{F}_{cyc}$ are the Coriolis and centrifugal terms, $\vec{G}$ are the gravitational forces and $\vec{F}_f$ are the friction forces on the PR. $\vec{F}_{ent}$ correspond to the forces applied by the patient to the PR and $\vec{\tau}$ stand for the active forces (actuated joints).

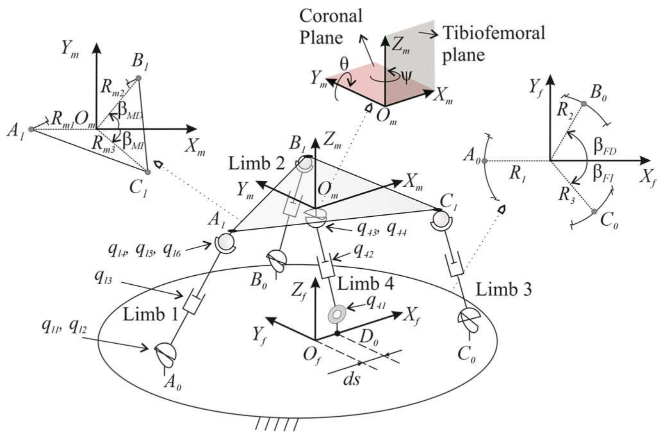


Fig. 1 Simplified view of 3UPS+RPU PR.

TABLE I
3UPS+RPU CONFIGURATION

Geometric Parameters		
R_1 (m)	R_2 (m)	R_3 (m)
0.4	0.4	0.4
R_{m1} (m)	R_{m2} (m)	R_{m3} (m)
0.3	0.3	0.3
β_{FD} (°)	β_{FI} (°)	d_s (m)
90	45	0.15
β_{MD} (°)	β_{MI} (°)	
50	90	

B. A linear dynamic model of the 4-DOF PR for identification

The 3UPS+RPU PR is a knee rehabilitation and diagnosis robot that works at $0.02\frac{m}{s}$ for translations and $0.03\frac{rad}{s}$ for rotations. This fact makes the inertial, centrifugal and Coriolis terms of the dynamic behavior insignificant. In addition, the patient forces are external to the PR, so they must be avoided to identify the inertial parameters of the PR, reducing the dynamic model to:

$$\vec{G}_{F\times1} + \vec{F}_{f_{F\times1}} = \vec{\tau}_{F\times1} \qquad (2)$$

Rewriting the gravitational term concerning the gravitational parameters $\vec{\Phi}_G$, the expression (2) becomes:

$$K_G \cdot \vec{\Phi}_G + \vec{F}_{f_{F\times1}} = \vec{\tau}_{F\times1} \qquad (3)$$

The $\vec{\Phi}_G$ collects the n parameters, which are the mass (m) and the first inertia moment ($[mx_G \quad my_G \quad mz_G]^T$) of all rigid bodies. An example of the first moment of the stem and cylinder of each actuator is $m_{ij}x_{G_{ij}}$ $i = 1..4, j = 1..2$, and for the mobile platform is $m_m x_{G_m}$.

The identification process for $\vec{\Phi}_G$ requires an overdetermined system achieved by applying (3) to N_{pts} different configurations of the PR under study, as follows:

$$W_G \cdot \vec{\Phi}_G = \vec{\mathrm{T}} \tag{4}$$

With

$$W_G = \begin{bmatrix} K_{G1} \\ K_{G2} \\ \vdots \\ K_{G_{N_{pts}}} \end{bmatrix}, \vec{\mathrm{T}} = \begin{bmatrix} \vec{\tau}_1 - \vec{F}_{f_1} \\ \vec{\tau}_2 - \vec{F}_{f_2} \\ \vdots \\ \vec{\tau}_{N_{pts}} - \vec{F}_{f_{N_{pts}}} \end{bmatrix} \tag{5}$$

C. Base parameter identification modifying SVD process

The SVD [20] applied to the W_G matrix generates the following equivalent multiplication involving submatrices:

$$W_{G_{m\times n}} = U_{m\times m} \cdot S_{m\times n} \cdot V_{n\times n}^T \tag{6}$$

Considering the rank (r) of W_G, the matrix $V_{n\times n}^T$ can be separated as:

$$V_{n\times n}^T = \begin{bmatrix} V_{1_{n\times r}} & V_{2_{n\times(n-r)}} \end{bmatrix} \tag{7}$$

Gautier at [13] introduces a permutation matrix (P) in Equation (4) to determine a compact system where the components of $\vec{\Phi}_G$ are linearly combined to determine a set of base parameter ($\vec{\Phi}_{base}$), given by:

$$\vec{\mathrm{T}} = W_G \cdot P \cdot P^T \cdot \vec{\Phi}_G = W_{G1} \cdot \vec{\Phi}_{base} \tag{8}$$

The matrix W_{G1} is a submatrix extracted from W_G as follows:

$$W_G \cdot P = \begin{bmatrix} W_{G1_{mxr}} & W_{G2_{mx(n-r)}} \end{bmatrix} \tag{9}$$

The $\vec{\Phi}_{base}$ are defined by:

$$\vec{\Phi}_{base} = \vec{\Phi}_{G1} - V_{21} \cdot (V_{22})^{-1} \cdot \vec{\Phi}_{G2} \tag{10}$$

with

$$P^T \cdot \vec{\Phi}_G = \begin{bmatrix} \vec{\Phi}_{G1_{1xr}} & \vec{\Phi}_{G1_{1x(n-r)}} \end{bmatrix}^T \tag{11}$$

The matrix P is found by combining rows of V_2 to generate two matrices V_{21} and V_{22}, where V_{22} must be non-singular. The original method given by Gautier [13] evaluates the whole set of combinations given by $\binom{n}{n-r}$ to look for a suitable P matrix, and to generate a non-singular matrix V_{22}, takes lot of computation resources. This research proposes a new method to select the best P matrix based on making subgroups of the combinations. With the new approach, several iterations are run, but only $\binom{n^*}{r}$ combinations are computed in each one, holding the elements that minimize the regressor matrix's condition number.

Firstly, this method decreases the number of rows available to take combinations from n to $n^* = (1+\alpha) * r$, where n^* is an integer and α is the coefficient factor ($0 < \alpha \leq 0.5$). In the first running, the group of row indices ($\vec{E}_G$) is formed by the first n^* row indices of V_2 stored in $\vec{E}$. The sub-combinations $\binom{n^*}{n-r}$ of $\vec{E}_G$ are calculated and stored as rows of the M_c. Then, V_{22} is defined by extracting the rows from V_2 according to the elements stored in each row of the M_c. If the V_{22} is a full rank matrix with a condition number less than the fixed limited ($cond_{limit}$), the sub-combination is stored at M_{vc} and the condition number of V_{22} is stored at C_{vc}. The rest of the sub-combinations are stored at M_{nc} and the condition number of V_{22} is stored at C_{nc}. At the final row of M_c, the M_{wc} matrix is defined as the 50% of worst sub-combinations in M_{nc} selected based on the condition numbers at C_{nc}. The most frequent element in M_{wc} defines the row index in $\vec{E}_G$ to remove, and a different row index from $\vec{E}$ is added to the $\vec{E}_G$. Finally, the combination in M_{vc} related with the minimum condition number in C_{vc} define the best permutation matrix P.

The proposed SVD process with the subgroup combination by statistical criterion does not evaluate every possible combination of matrix P, resulting in efficient use of computational resources.

In the 3UPS+RPU PR case, the SVD process proposed by Gautier takes 30 minutes to evaluate every possible combination (155.12 million) for the matrix P. In contrast, with the subgroup combination using statistical criterion, where $cond_{limit} = 2000$, $\alpha = 0.25$, $n^* = 19$ (3876 combinations per group) takes just 7 seconds to find the best combination for matrix P. Both tests were performed in a desktop computer with Core i7 2.60GHz processor and 16GB RAM memory.

The new subgroup combination method to find the P matrix is described by the following pseudocode.

PARAMETERS:
Limit of condition number $cond_{limit}$
Group rising factor α
Number of elements for reduced combination n^*
Vector with elements to combine $\vec{E}$
INPUTS:
Matrix V_2 that comes from SVD process, expression (7)
Number of columns of W_G (n)
Rank of W_G (r)
OUTPUTS:
Transpose permutation matrix (P^T)
INITIALIZATION:
$cond_{limit} = 2000$
$\alpha = 0.25$
$n^* = (1+\alpha) * r$
$le = n^*$
$\vec{E} = [1 \quad 2 \quad \dots \quad n]$
Define $\vec{E}_G$ vector with the first n^* elements of $\vec{E}$
M_{nc} empty matrix
M_{vc} empty matrix
C_{nc} empty vector
C_{vc} empty vector
P^T eye matrix, with $n \times n$ size

BEGIN
WHILE $le \leq n$
Define M_c matrix, where each row is a combination of $(n-r)$ different elements from $\vec{E}_G$

FOR i = 1:number of rows of M_c **DO**
Define V_{22} as a sub-matrix of V_2 where the rows are selected by the elements in the row i of M_c
IF $rank(V_{22}) = (n - r)$
IF condition number of $V_{22} < cond_{limit}$
Add the elements of the row i of M_c to M_{vc}
Add the condition number of V_{22} to C_{vc}
END
ELSE
Add the elements of the row i of M_c to M_{nc}
Add the condition number of V_{22} to C_{nc}
ENDIF
END
IF $le < n$
$M_{wc} = M_{nc}$
Sort the rows of M_{wc} in ascending order according to the related condition number stored in C_{nc}
Delete the first half of M_{wc}
Define we as the most frequent element inside of M_{wc}
Delete the element of $\vec{E}_G$ equal to we
Add the $le + 1$ element in $\vec{E}$ to $\vec{E}_G$
Clear matrix M_{nc}
Clear vector C_{nc}
END
$le = le + 1$
END
Sort the rows of M_{vc} in ascending order according to the related condition number stored in C_{vc}
Sort the last $n - r$ rows of P^T according to the elements in the first row of M_{vc}
END

D. *Identification guideline for the 3UPS+RPU PR*

The identification of dynamic parameters could be developed directly or indirectly. The direct method identifies the inertial and friction parameters simultaneously using a unique experiment. The indirect method uses different experiments to identify inertial and friction parameters sequentially [21].

This work selected the indirect identification of $\vec{\Phi}_{base}$ A linear friction model has been previously identified because the friction parameters identification depends on factors like the joint surface condition, getting hard to identify rigid body parameters [10]. The Coulomb-viscous friction model is selected because the robot under study is moved at low velocity, and linear actuators power it.

The accuracy of the base parameter's identification depends on the dynamic excitation produced by the designed trajectories. Due to the PR under study is designed for knee rehabilitation and has singularities inside its workspace [8] the identification trajectories are nine. The nine trajectories are specifically developed for the gravitational base parameter identification of the 3UPS+RPU PR. The nine identification trajectories are a combination of three fundamental knee movements: i) extension- flexion of the knee, ii) external-internal rotation knee, and iii) flexion of the hip [17], [18].

The identification performed in simulation calculates $\vec{T}$ using the inertial parameters defined for a tridimensional virtual model of the 4-DOF PR in SolidWorks. A random noise of $\pm 12N$ is added to $\vec{T}$, to make the simulation more realistic.

In the actual PR, the identification process is developed after an experiment of 20 minutes where the mobile platform raises and lowers to set the friction effect in nominal condition of work. The actual $\vec{T}$ is calculated using the $\vec{\tau}$ developed by the controller and the friction forces estimated by the linear Coulomb-viscous model.

III. Results and Discussion

A. *Influence of the gravitational term on the dynamic behavior of the 3UPS+RPU PR*

Fig. 2 shows the contribution of the $\vec{F}_{in}$, and the $\vec{F}_{cyc}$ to $\vec{\tau}$ and Fig. 3 shows the contribution of $\vec{G}$ and $\vec{F}_f$ to $\vec{\tau}$ for a knee rehabilitation trajectory developed for the 4-DOF PR under study (actuators on limbs 1 and 4). Based on these figures, it is verified that the relevant terms of the dynamic behavior of the 3UPS+RPU PR are the gravitational and friction terms.

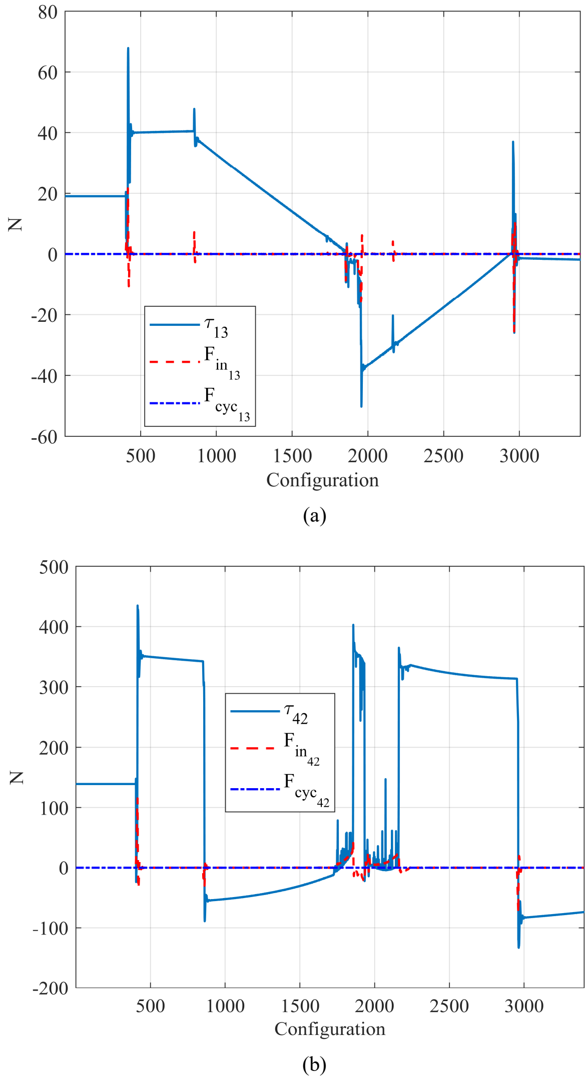


Fig. 2 Inertial, Centrifugal, and Coriolis forces for limb (a) 1 and (b) 4.

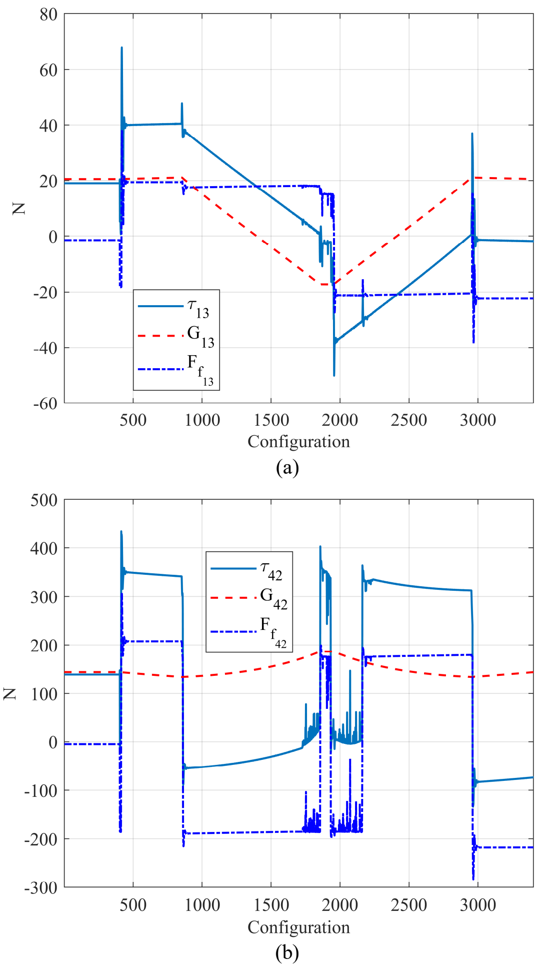


Fig. 3 Gravitational and friction forces for limb (a) 1 and (b) 4.

B. Base parameter identification in the simulated 3UPS+RPU PR

After applying the SVD procedure with subgroup combination using statistical criterion, 15 base parameters with a physical sense have been identified (Table II).

TABLE II
BASE PARAMETER IDENTIFIED FOR 3UPS+RPU PR

Nº	Base Parameter
1	$m_{12} + m_{22} + m_{32} + m_{42} + m_m$
2	$m_{11}x_{G_{11}} + m_{12}x_{G_{12}}$
3	$m_{21}x_{G_{21}} + m_{22}x_{G_{22}}$
4	$m_{11}z_{G_{11}} + m_{12}z_{G_{12}}$
5	$m_{21}z_{G_{21}} + m_{22}z_{G_{22}}$
6	$m_{31}z_{G_{31}} + m_{32}z_{G_{32}}$
7	$m_{31}x_{G_{31}} + m_{32}x_{G_{32}}$
8	$m_{42}y_{G_{42}} - m_{41}z_{G_{41}}$
9	$0.1928m_{22} - 0.3m_{12} + m_m x_{G_m}$
10	$0.2298m_{22} - 0.3m_{32} + m_m y_{G_m}$
11	$m_m z_{G_m}$
12	$m_{42}x_{G_{42}} - m_{41}x_{G_{41}}$
13	$m_{11}y_{G_{11}} + m_{12}y_{G_{12}}$
14	$m_{21}y_{G_{21}} + m_{22}y_{G_{22}}$
15	$m_{31}y_{G_{31}} + m_{32}y_{G_{32}}$

In addition, to verify that the new modification of the SVD procedure achieved feasible base parameters, the inertia transfer procedure is applied to the PR under study. Considering the mobile platform as the rigid body upon which inertial parameters are projected, the symbolic results are equal to their presented in Table II. The symbolic equivalency of the base parameters 9 and 10 are presented in Table III.

TABLE III
INERTIA TRANSFER IDENTIFICATION FOR BASE PARAMETER 9 AND 10

Nº	Inertia Transfer
9	$R_{m2}\cos(\beta_{MD})\, m_{22} - R_{m1}m_{12} + m_m x_{G_m}$
10	$R_{m2}\sin(\beta_{MD})\, m_{22} - R_{m3}\sin(\beta_{MI})\, m_{32} + m_m y_{G_m}$

For a non-singular rehabilitation trajectory, the matrix W_G has a condition number of 8.64×10^{33}, which is too high. The 15 base parameters lower down the condition number of the matrix W_1 to 646.6, which is still high.

The architecture of 3UPS+RPU PR has been optimized [8] in order to reduce the condition number of W_1. This work deletes the base parameters with less dynamic influence on $\vec{\tau}$. The forces $\vec{\tau}$ are mainly influenced (95%) by the first 8 base parameters (see Fig. 4), reducing the condition number of simulation experiments to 73.5.

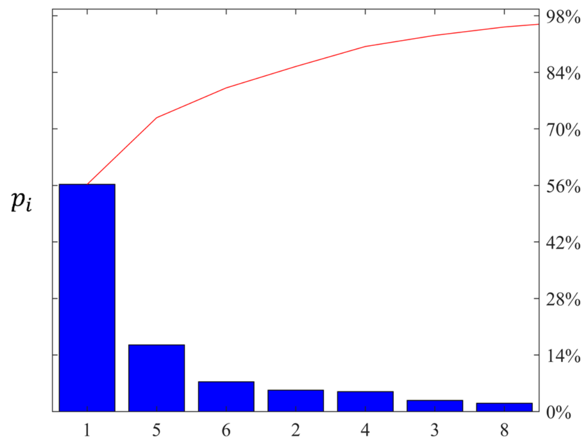


Fig. 4 Dynamic influence of the identified Base parameters.

The blue bars indicate how much accuracy (in %) is gained in the estimation of $\vec{\tau}$ when including each base parameter as determined in Equation (12), in descending order of influence.

$$p_i = \frac{\sqrt{\vec{A}^T \cdot \vec{A}}}{\sqrt{\vec{T}^T \cdot \vec{T}}} \cdot 100 \qquad (12)$$

where

$$\vec{A} = W_{G_{m\times i}} \cdot \vec{\Phi}_{G_{i\times 1}} \qquad i = 1 \dots 8 \qquad (13)$$

Table IV shows the result of the gravitational identification performed in simulation after selecting the eight most influential base parameters. It is important to mention that the inertial parameters of the PR under study are taken from a tridimensional mechanism designed according to the actual PR.

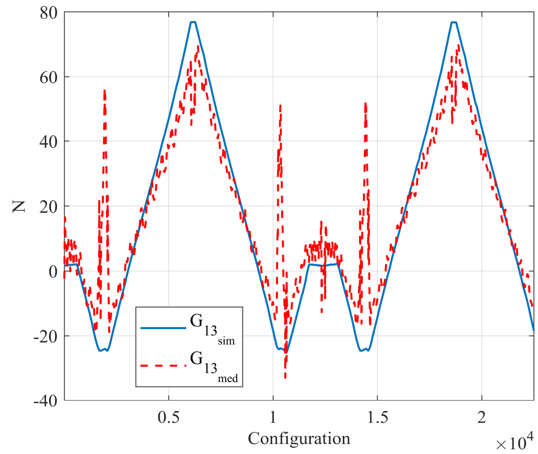
TABLE IV

BASE PARAMETER VALUES FOR SIMULATED PR

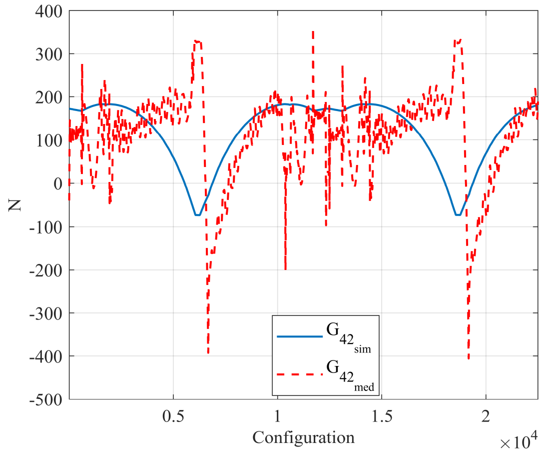
Nº	Base Parameter	Value
1	$m_{12}+m_{22}+m_{32}+m_{42}+m_m$	14.12
2	$m_{11}x_{G_{11}}+m_{12}x_{G_{12}}$	-0.35
3	$m_{21}x_{G_{21}}+m_{22}x_{G_{22}}$	0.17
4	$m_{11}z_{G_{11}}+m_{12}z_{G_{12}}$	1.58
5	$m_{21}z_{G_{21}}+m_{22}z_{G_{22}}$	1.41
6	$m_{31}z_{G_{31}}+m_{32}z_{G_{32}}$	1.40
7	$m_{31}x_{G_{31}}+m_{32}x_{G_{32}}$	0.15
8	$m_{42}y_{G_{42}}-m_{41}z_{G_{41}}$	-0.94

C. Base parameter identification in the actual 3UPS+RPU PR

The actual gravitational forces for limb 1 and limb 4 are presented in Fig. 5a and Fig. 5b, respectively. For limb 1 the difference between simulation (G_{*sim}) and real forces (G_{*med}) is low, where $*$ identifies the actuator analysed. However, the gravitational forces appearing at limb 4 are ten times greater than those at the other limbs, increasing the error between G_{*sim} and G_{*med}. This difference is reflected in the base parameter identification on the actual 3UPS+RPU PR.



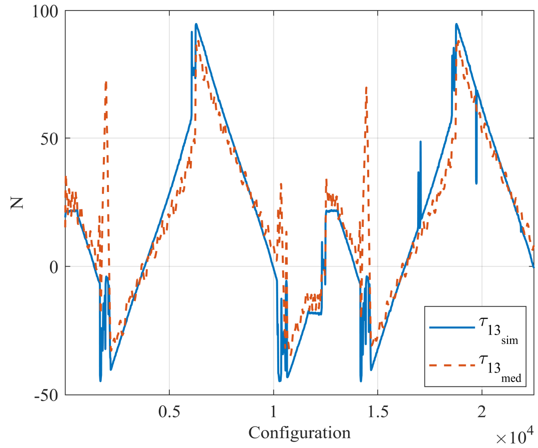
(a)



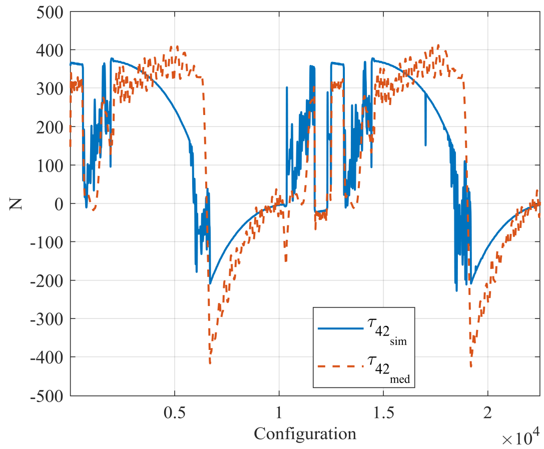
(b)

Fig. 5 Simulated and actual gravitational forces for the 3UPS+RPU PR in limb (a) 1 and (b) 4.

Fig. 6a shows the simulated active forces composed of the compact gravitational term and the friction force (τ_{*sim}), whereas Fig. 6b presents the actual active forces (τ_{*med}), for limbs 1 and 4. In this figure, it can be noted that the difference between the simulation and the actual experiment is acceptable.



(a)



(b)

Fig. 6 Total control action for the 3UPS+RPU PR in limb (a) 1 and (b) 4.

The results of the base parameters identification developed in the actual 3UPS+RPU PR are presented in Table V. The actual identification shows that the first base parameter is similar to the simulation identification with a 1.48 Kg of error, which represents a 10.48% of the base parameter obtained in simulation. The other seven are different due to the force difference presented in the gravitational term on limb 4.

TABLE V

BASE PARAMETER VALUES FOR ACTUAL PR

Nº	Base Parameter	Value
1	$m_{12}+m_{22}+m_{32}+m_{42}+m_m$	15.60
2	$m_{11}x_{G_{11}}+m_{12}x_{G_{12}}$	1.46
3	$m_{21}x_{G_{21}}+m_{22}x_{G_{22}}$	0.82
4	$m_{11}z_{G_{11}}+m_{12}z_{G_{12}}$	0.82
5	$m_{21}z_{G_{21}}+m_{22}z_{G_{22}}$	-0.89
6	$m_{31}z_{G_{31}}+m_{32}z_{G_{32}}$	-1.58
7	$m_{31}x_{G_{31}}+m_{32}x_{G_{32}}$	-0.37
8	$m_{42}y_{G_{42}}-m_{41}z_{G_{41}}$	-0.76

IV. Conclusion

For the 3UPS+RPU PR applied to knee rehabilitation, the dynamic behavior is mainly affected by the gravitational and friction forces. With an accurate friction model, the gravitational term is the most time-variant term of the dynamic behavior of the 3UPS+RPU PR.

Using statistical analysis, the subgroup combination procedure proposed to find the best permutation matrix in the SVD identification process reduces the computational cost while achieving gravitational base parameters with physical feasibility. In addition, the physical sense of the base gravitational parameter identified was verified by applying the inertial transfer identification to the PR under study.

The base parameter identification developed in the simulated and actual 3UPS+RPU PR using knee rehabilitation trajectories identifies the mass of the mobile platform as the most relevant time-variant parameter. It this study, the identification procedure has an error of 1.48 Kg (10.48%) in estimating the first base parameter.

The set of the eight gravitational base parameters with 95% of the dynamic behavior of the 3UPS+RPU PR, are the basis for developing adaptative controllers. Nowadays, the results of this research are being applied in hybrid controllers designed for the PR under study.

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