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Estimating Indonesian Complete Life Table and Fair Annual Pure Premium Range from Abridged Life Table with Bayesian Method and Bootstrapping

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Abstract— **Several Indonesian life insurance companies recently faced financial problems due to inadequate pricing and idealistic investment expectation. Growing market and insurtech implementation might lead to worse conditions in the future. The current mortality table and investment return assumption are too ideal, so more conservative assumptions are required to get a more reasonable annual pure premium range. This research estimated complete life tables from abridged life tables by truncated Heligman-Pollard and Makeham model, when a lognormal stochastic process estimated annual investment return. Parameters for mortality models and return distribution are estimated using Bayesian method with Metropolis-Hasting's algorithm. Data from the abridged life table was bootstrapped due to insufficient number for statistical parametric modeling. Good accuracy for estimated abridged mortality rates was reached by referring to the Mean Absolute Percentage Error (MAPE) metric for both males and females, also for the young ages group (new-born to twenty-nine years old) and old ages group (thirty to eighty-four years old). The parameters were satisfactory to estimate the complete life table and extrapolate annual mortality rates calculation until age ninety-nine. A log-normal distribution was found to fit the monthly inflation rates satisfactorily. Assuming that investment return is close to the inflation rates, the annual investment return is anticipated for both profitable and losing situations. Therefore, insurance companies can win the customers' decisions without compromising their financial stability.**

Keywords— **Investment return; life insurance; parametric mortality model.**

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I. INTRODUCTION

Indonesian insurance law mandates the formation of an institution to guarantee customers' policies' payment although their insurers are bankrupt. Its urgency increases after financial problems that hit two state-owned insurance companies, Asabri and Jiwasraya [1]. A reliable life table and investment return assumption are needed to determine whether premiums charged to customers are appropriate or not. The government also needs a reference to decide whether insurance companies have set profitable premium tariff or not and charge how much they have to pay to the policy guarantor institution.

Society of Actuaries of Indonesia published 4th Indonesian Mortality Table (TMI IV) based on an experience study from 52 domestic insurance companies' data during 2013 to 2017. Data on insureds who had passed the underwriting process to ensure that they were healthy and doing activities safely were used to form this table. In other words, TMI IV is a select mortality table and cannot be treated as a population mortality table.

In line with efforts to increase number of customers and widen insurtech implementation, the underwriting process will be loosened up. As we mentioned in the previous paragraph, TMI IV is not suitable to be generalized to represent the whole Indonesian population. Generalization from that table could result in adverse selection and too low premiums to be charged. Infant mortality rate and life expectancy calculation support the previous statement with numbers in Table I.

Without adequate data to build a complete life table provided by the government, we sought available third-party life tables that might be more reliable in representing the population. To the best of our knowledge, the best available table is the abridged life table for 2015 – 2020 period [2], [3]. The rates provided are more reasonable as it comes closer to the population estimate provided by the Indonesian Central

Bureau of Statistics, but annual mortality rates are still estimated.

TABLE I
INDONESIAN INFANT MORTALITY RATE AND LIFE EXPECTANCY

Metric	Source	Study period and population	Value
Infant mortality rate (q_0)	Indonesian Central Bureau of Statistics [4]	2017 (both sexes)	0.024
	TMI IV	2013 – 2017 (males)	0.00524
		2013 – 2017 (females)	0.00266
Newborn life expectancy (e_0)	Indonesian Central Bureau of Statistics [5]	2019 (males)	69.44 years
		2019 (females)	73.33 years
	TMI IV	2013 – 2017 (males)	78.40 years
		2013 – 2017 (females)	81.95 years

Another problem regarding premium determination is an idealistic high investment return assumption set to lower the premiums. For example, many Indonesian life insurance companies charge an annual premium equal to ten times that of a monthly paid premium. By assuming a uniform distribution of deaths for fractional ages to be applied to the TMI IV, we found that they have to realize an annual investment return of at least 36.33% in order to get an equal value of money. It could lead to portfolio formation of junk financial market products in order to reach the high return target, but the risk implied is also too high. Although long-term historical trend supports this assumption, the market contains its uncertainty and is going to a long-term equilibrium state. Compared to the data obtained from Rosha [6] and BPS [7], this assumption had never been at least equaled by the year-on-year return obtained by Jakarta Composite Index (JCI) in the last nine full years (2011-2019). It is even worse than the inflation rate beating the increase rate of JCI five times in nine occurrences, including for the last two years.

From a theoretical point of view [8], industry competitiveness tends to bring companies down into zero-profit long-run equilibrium. In this situation, revenues only cover industrial costs, implying that industrial growth should not be more than increase of their costs. Therefore, we argue that a more reasonable option for this study is assuming the return is equal to the national inflation rate.

This study aims to produce a fair annual pure premium range estimation for Indonesian life insurance. Two main components that are required in the pure premium range are the mortality rates and investment returns, which need to be estimated beforehand. Thus, this study consists of three main processes: estimation of Indonesian abridged mortality rates, estimation of investment return, and estimation of fair annual pure premium range as the ultimate result.

This study's novelties are in optimizing the abridged life table as a substitute for the unavailable Indonesian population's recent complete life table for estimation of Indonesian mortality rates and the procedures developed to overcome the limitations of the abridged life table in the estimation process. While other studies fit one model for all age ranges, we argue that different patterns of mortality rates might occur in different age ranges. Thus, upon empirical assessment of the mortality rates, we propose modifying the Heligman-Pollard model, namely the truncated Helligman-Pollard model. Furthermore, we introduced the so-called intermediary parameters to overcome the complexity of constructing prior distributions for the modified model. And finally, we define the best- and worst-estimate of q_x, the mortality rate at age x, as approximations for the complex non-closed posterior distributions. These are explained further in the subsequent sections.

II. Materials and Method

This section explains our objectives and compares them with previous related works. We also describe our data and explain the methodology and mathematical models to fit the data.

A. Objectives and Related Works

The first objective of this study is to estimate the Indonesian complete life table based on United Nations' abridged life table for the period 2015-2020 with the assumption of compliance with a parametric mortality model. Since we are not confident enough that the samples involved in estimating that the abridged life table fully represents the population, we implement Bayesian method to result in a more reasonable range of estimated annual mortality rates. This is due to the nature of the Bayesian approach, where the inference is not solely based on data but also incorporates experts' judgment through the prior density. In this way, an optimal result is still obtained even under low-qualified data [9]–[12].

The second objective of this study is to estimate a reasonable range of investment returns by considering the probability of loss. This approach is superior to mitigating risks coming from unexpected negative situations, such as disease pandemics, economic slowdown, political chaos, and trade war. When the value of an investment portfolio plummets, insurance companies still have a reasonable buffer to protect themselves from investment loss. This investment return range is needed in pricing, reserving, and scenario testing.

A study for estimating a complete life table from an abridged life table was initiated by Heligman and Pollard [13]. They proposed using the full Heligman-Pollard mortality model and estimated the parameters using the Bayesian method. This approach is considered superior to the standard least squares method to avoid overparameterization and the possibility of illogical parameter estimates. The statistical inference could also be derived for complete and incomplete life tables. They tried to estimate population size in every age group, given the total population size. However, it is hard to verify the results because mortality tables are published without the underlying data to construct them. Prior distributions in their study were also decided based on focus group discussion of mathematical researchers and not involving professionals with a demographic or medical degree, rather than using the results of previous studies. Later, we found that their 98% confidence interval for prior distributions of A, B, C, and E is too small, but their domain for parameter F is too wide at the same time. It is also strange that their lower limit exceeds the upper limit for mortality rates of toddlers (see Fig. 5 in [13]).

A study on Malaysian population [14] used the interpolation method without assuming any parametric mortality model, but the results only pointed to estimations rather than interval estimations. Li went one step ahead with fitting simplified Gompertz model for old ages but still based on the interpolation from young ages data [15]. The results were also in the form of point estimations. However, there is no reason we should expect a point estimate from a given sample to be exactly equal to the condition of the population, although we have large samples [16]. It is preferable to determine an interval estimate which we would expect to find the value.

B. Data Description

The first data to estimate the complete life table is the Indonesian abridged life table for the 2015 – 2020 period, each male and female population, which was published as a part of World Population Prospects 2019 and downloaded from the United Nations [2], [3]. In this research, we treated q_0, ${}_4q_1$, ${}_5q_5$, ${}_5q_{10}$, ${}_5q_{15}$, ${}_5q_{20}$, ${}_5q_{25}$, ${}_5q_{30}$, ${}_5q_{35}$, ${}_5q_{40}$, ${}_5q_{45}$, ${}_5q_{50}$, ${}_5q_{55}$, ${}_5q_{60}$, ${}_5q_{65}$, ${}_5q_{70}$, ${}_5q_{75}$, and ${}_5q_{80}$ as data to fit the mortality model. While ${}_5q_{85}$, ${}_5q_{90}$, and ${}_5q_{95}$ were considered to evaluate our model ability in extrapolation.

The second data to estimate a reasonable range of investment return is Indonesian monthly inflation rates for January 2006– December 2019 [17]. It consists of 168 records of monthly inflation rate data. Considering some previous studies [18]–[27], this period is adequate to capture the ups and downs of Indonesian economy. Indonesia faced long-term effects of Great Moderation of Chinese Economics, great recession of global economics and also bankruptcy of Century Bank in 2008, and enactment start of ASEAN-China Free Trade Agreement (ACFTA) in 2010. Indonesia also experienced significantly increased of subsidized fuel price in 2013, change of national leadership in 2014, Brexit process since 2016, and Policy Normalization by The Federal Open Market Committee (FOMC) of United States since 2017. Trade war between United States and China since 2018, geopolitical issues in The Middle East since 2019, and the beginning of COVID-19 outbreak in the end of 2019 also influenced Indonesian economy. The subsequent sections discuss the method and mathematical formulation constructed in this study. Summary of the procedures is depicted in Fig.1.

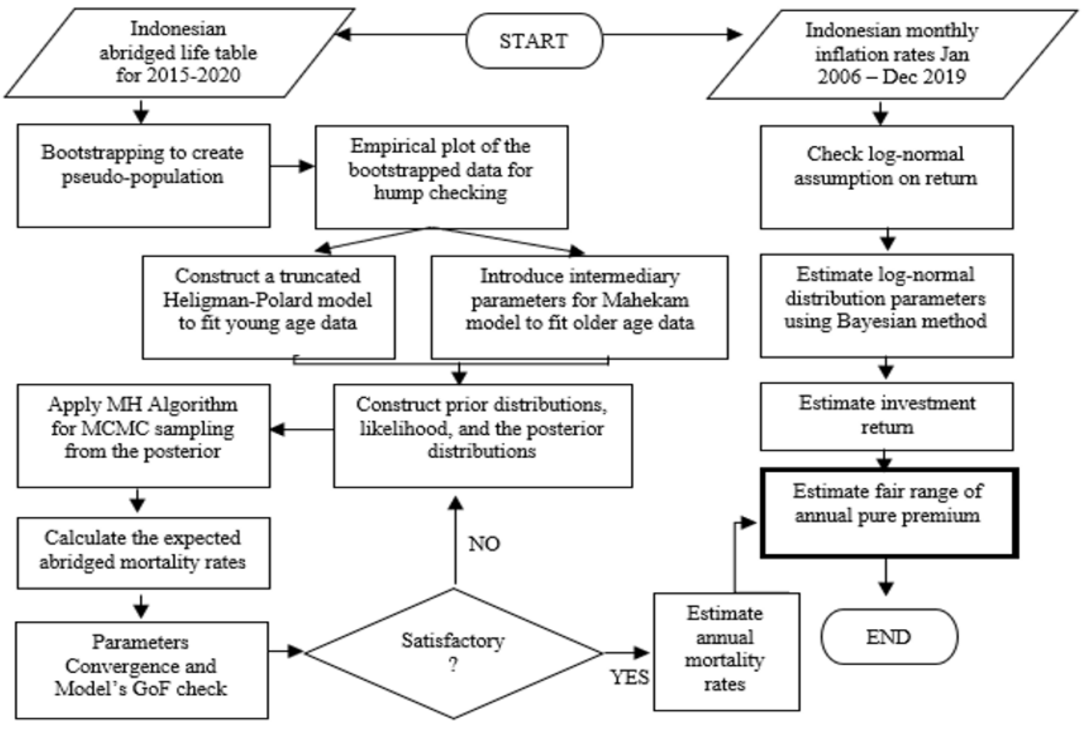

Fig. 1 Flowchart of the research method.

C. Bootstrapping and Mortality Modelling

Two parametric models that are considered to produce good fits on annual mortality rates for all ages are Heligman-Pollard model [13] and Siler model [28]. Siler model is more compact than Heligman-Pollard model but fails to capture accident humps. While, Heligman-Pollard model ensures a good fit with data but is often overparameterized [29]. Heligman-Pollard model could capture a hump and dynamic trend, i.e., a repeated pattern of rising and fall of the mortality trend. While in the Siler model, there are no humps nor a dynamic trend, and instead, the mortality rates are directly increasing after ending its decreasing movement.

Abridged life tables contain a small amount of data. Although the estimated values produced by Bayesian method are considerably better than those from the frequentist method in a small sample size, the estimates are sensitive to prior distribution specifications, and inappropriate handling can lead to worse estimation than that of frequentist methods [11]. In this research, we handle a small sample size by generating more samples based on the data with the bootstrap procedure as described below.

- For every age group, a pseudo population of size 100,000 are generated. The probability of death in the data is multiplied by 105 and then floored to get the number of deaths in the interval, and the rest should be survivors. It can be accepted historically and mathematically for these two reasons.
  Historically, mortality rates are expressed in the ratio per number of populations. The number used often is 1,000 and the example institution used it is [30]. Another number that is often used is 100,000 with Global Health Observatory as an example [31].
  Mathematically, Agresti and Min [32] suggested that to have probability variation of binomial distribution, the minimum of successes and fails are ≥15. In this case, we define success events as deaths and failed events for the survivors. For all age groups, our procedure satisfied the condition. The minimum number of successes in an age group is 203, and the minimum number of fails in an age group is 50,626.
  We chose 100,000 for all age groups, and this practice is only for creating bigger sample sizes of the mortality rates. The supposedly different proportions for each age group are already represented by the probability of death in the data. Some may wonder if the population at younger ages will be more than that at older ages. We would have further consideration for this point in our mathematical model.
- From the pseudo populations, 100,000 samples of individuals are taken with repetition. The proportion of deaths were calculated and saved as final samples from bootstrap procedure.
- For every age group, 100 final samples of mortality rates are gathered.

Observing the pattern of bootstrapped abridged mortality rates, there is a tendency of slowing mortality growth in age interval (20, 25) for males and (15, 20) for females. Empirically, we believe that there is accident hump in both mortality patterns and the evidence is shown in Fig. 2. for the male population. The Heligman-Pollard model is more suitable for this case than the Siler model.

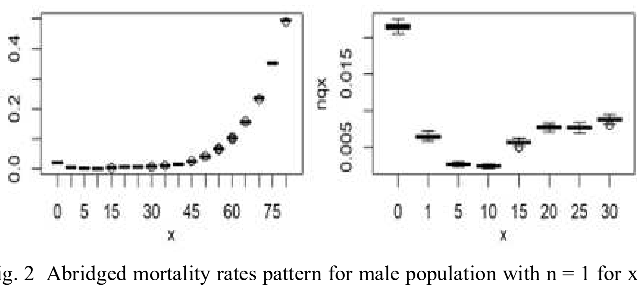

Fig. 2 Abridged mortality rates pattern for male population with n = 1 for x = 0, n = 4 for x = 1, and n = 5 for other x values. Left graph is for age 0 – 84, when right graph is restricted for age 0 – 34.

Instead of directly applying the full Heligman-Pollard model to the data, this research splits the fitting process into two phases. The first phase tests the presence of accident hump in young ages, and the second phase is learning exponential growth of mortality rate in old age. The initial Heligman-Pollard model was defined such that A and B determine mortality rate for babies, C determines mortality rate decline for childhood, D, E, and F explain accident hump phenomenon for adolescence, and G and H determine old ages mortality f. The model is written as (1), with q_x representing the annual mortality rate for an individual with exact age at x.

$$\frac{q_x}{1-q_x} = A^{(x+B)^C} + D\exp\left(-E\left(\log\left(\frac{x}{F}\right)\right)^2\right) + GH^x \quad (1)$$

Following Alexopoulos et al [33], logical range of values for each parameter are as the following: $A \in (0,1)$, $B \in (0,1)$, $C \in (0,1)$, $D \in (0,1)$, $E > 0$, $F \in (10,40)$, $G \in (0,1)$, and $H > 0$. According to Wunsch et al. [34], A is a crude approximation to q_1, B is a crude approximation of the difference between q_0 and q_1, C reflects the speed of childhood mortality rate decline, D measures hump severity, E measures hump width, and F is mean of age to have mortality rate surge due to accident, respectively. G and H parameters are derived from the Gompertz mortality model. By looking at all the definitions, we determined that the ranges given are plausible and proceed further with them.

1) Mortality Modelling for Young Ages: We considered that old mortality has a small correlation with young mortality. We also assumed that mortality rates must be increasing for ages over thirty. Therefore, we could truncate Heligman-Pollard model by restricting implementation only for $x \leq 29$ and removing two parameters, G and H. We proposed the truncated model to further check whether the hump happened or not. If $F \geq 30$, there is no hump, and the mortality rate is surely increasing once an individual leaves the childhood phase. If $F < 30$, there is a hump, and we assume that mortality rates will not increase before the individual leaves late adolescence phase. Our formula for estimating mortality rates for young ages as the following:

$$\frac{q_x}{1-q_x} = A^{(x+B)^C} + D\exp\left(-E\left(\log\left(\frac{x}{F}\right)\right)^2\right) \quad (2)$$

Our bootstrapped data was considered to be enough. Following a recommendation in some previous studies [35]–[38], we had 700 observations, and it is more than their rule of thumb that the required data varies around 300 to 350 observations. The data size is also more than enough compared to the number of parameters in the formula; as [39] recommended, an ideal sample size ratio to the number of parameters is 20:1.

We consider prior specification studies [40], [41]. Since their results vary and we do not have enough data to determine the exact form of prior distributions, we robustly defined them for our research, so the probability for values observed in prior research lie between 70% and 99.5% (except for parameter F) with their domains match the definition in Alexopoulos et al. [33]. Therefore, selected prior distributions are two-parameters beta, uniform, and gamma with parameterization [42].

Since our data is in the form of probabilities and the exact population size to construct the abridged life table is unknown, beta distribution is chosen for the sampling model. Beta distribution is considered to be more suitable than binomial or Poisson distribution due to unknown population size and flexibility of determining mean-variance relationship. We denoted Q_Y as a set of seven abridged mortality rates for young ages that contains q_0, ${}_4q_1$, ${}_5q_5$, ${}_5q_{10}$, ${}_5q_{15}$, ${}_5q_{20}$, and ${}_5q_{25}$. We expected that the mean of sampling model for each age group is equal to the mean of bootstrapped samples from [2] and [3]. Large variances are expected to represent our low trust in the data without computational problems during iteration. The low trust is considered because of the possibility of inadequate sample size and variation of sample characteristics, inappropriate sampling technique, and measurement errors. If $q_{Y,i}$ represents mortality rate for i^{th} age group, then:

$$(Q_{Y,i}|A, B, C, D, E, F) \sim \text{beta}\left(1, \frac{1-q_{Y,i}}{q_{Y,I}}\right) \quad (3)$$

The truncated Heligman-Pollard model in (2) could only calculate annual mortality rates. Yet, we need the abridged mortality rates for the calculation in the sampling model. Therefore, except for $q_{Y,1}$ (which represents q_0), the abridged mortality rates should be calculated using formula (4).

$$ {}_nq_x = \sum_{k=0}^{n-1} {}_{k|}q_x \quad (4)$$

By assuming every pair of abridged mortality rates is independently distributed, the joint distribution in the sampling model can be expressed as a product of each observation density. By multiplying prior densities and sampling model, we obtained our posterior, ignoring the normalizing constant,

$$\text{Posterior} \propto a^{2.10212}(1-a)^{42.6376}c^{4.50156}(1-c)^{13.7807}$$
$$d^{4.87727}(1-d)^{6312.76}e^{4.79149}\exp\left(-\frac{e}{2.14625}\right)$$
$$\prod_{i=1}^{7}\left(\frac{\Gamma\left(\frac{1}{q_{Y,i}}\right)}{\Gamma\left(\frac{1-q_{Y,i}}{q_{Y,i}}\right)}(q_{Y,i})^{\frac{1}{q_{Y,i}}-1}(1-q_{Y,i})^{\frac{1}{q_{Y,i}}-2}\right) \quad (5)$$

The posterior which was described in (5) made us difficult to obtain full posterior conditional distribution for every parameter, so any analytical calculation, simple Monte-Carlo simulation, or Gibbs sampling algorithm could not be implemented. Therefore, a more sophisticated Metropolis-Hastings (MH) algorithm, as defined in Section 10.4.1 [43] was implemented to estimate the posterior distributions of each parameter.

We were inspired by the random jump proposing method [44], but our parameters' ranges could not satisfy the domain of normal distribution. Thus, we considered other distributions that were constructed by transforming a normal distribution, except for parameter F with values ranging in a closed interval. Once again, except for parameter F, we aimed the expected value or mode for the next proposal equals the current iteration value. The proposal variations were determined by looking at each parameter's 95% confidence interval width, depending on which one is easier to calculate in closed form.

Since it was hard to obtain posterior distribution of annual mortality rates (q_x), we estimated the expected value of these quantities by substituting the expected value of every parameter into (2). Annual mortality rates are strictly increasing function of A, strictly decreasing function of B, increasing function of C (for x = 0). However, it also decreasing function of C (for $x \geq 1$), strictly increasing function of D, and strictly decreasing function of E. Relation of q_x and F is quite complicated, it is increasing function of F if $x > F$, and otherwise when $x < F$. Hence, for constructing the range, we defined best and worst estimate generally based on matching 2.5-percentile and 97.5-percentile with the characteristic of mortality rate's function.

2) Mortality Modelling for Old Ages: Makeham model is often considered for industrial practitioners to estimate mortality rates for old ages with its extrapolation ability. Two life tables that were constructed by implementing Makeham model are the Japanese Standard Mortality Table (2018) [45] and the 4th Indonesian Mortality Table (2018). According to Bowers et al. [46] and Dickson et al. [47], Makeham model is defined by expressing the force of mortality at age x (μ_x) as:

$$\mu_x = A + BC^x \quad (6)$$

Domain for the parameters are $\{(A, B, C) \mid A > -B, B > 0, C \geq 1\}$. According to [48], parameter values are commonly contained in this set: $\{(A, B, C) \mid 0.001 < A < 0.003, 10^{-6} < B < 10^{-3}, 1.08 < C < 1.12\}$. Usually, good fit occurs for the 30 to 80 years old population when mortality rates are increasing exponentially in age. After that, the growth rate can still be the same or a bit decreasing, which is assumed by the 3rd Heligman-Pollard model as written [40] and concluded from the explanation by Bebbington et al. [49]. Similar to Section II.C.1, our bootstrapped data was considered enough by following recommendations [35]–[38].

Regarding previous explanation in Section II.B, we used ${}_5q_o$ data to fit the model with O = {30, 35, 40, …, 75, 80} and tried to extrapolate the 5-year mortality rates until x = 95. If we accurately estimated them, we could proceed to calculate annual mortality rates for age intervals [30, 99]. Considering previous research [45], [50], also looking at G and H values in prior research with the full Heligman-Pollard model (which have similar meanings respective to B and C in (6)), we tried to maximize the probability of observed values in prior research.

However, finding a simple distribution for representing A and C was not easy. Therefore, we did not directly estimate A and C, but incorporated two intermediary parameters. First, we knew that A > -B and their general values are very small. So, A + B must be a positive number that values very small. Instead of directly estimating A, we define a new parameter $\alpha = A + B$. Second, C is slightly over 1, so C – 1 is a little over 0. Rather than directly estimating C, we would prefer to define a new parameter $\varsigma = C - 1$. Therefore, selected prior distributions are exponential and gamma with parameterization [51].

We would follow a similar fashion when defining our sampling model for young ages. Now we denoted Q_O as a set of abridged mortality rates for old ages that contain ${}_5q_{30}$, ${}_5q_{35}$, ${}_5q_{40}$, …, until ${}_5q_{80}$. We expected the mean of sampling model for each age group is equal to the mean of bootstrapped data. We also accommodate the fact that the population size decreases in terms of age. Furthermore, the variances were maximized to represent our low trust in the data without computational problems during iteration. Thus, if $q_{O,i}$ represents mortality rate for i^{th} age group, then:

$$(Q_{O,i}|\alpha, B, \varsigma) \sim \text{beta}\left(1, \frac{1-q_{O,i}}{q_{O,I}}\right) \quad (7)$$

Notice that we use similar sampling model for young-aged and old-aged population. If the abridged mortality rate is q, mean and variance of the sampling model equals q and $\frac{q(1-q)}{\frac{1+q}{q}}$. Since q is probability of individual death, Q is fit to proportion data of the population, actually, the values of both q and Q are random, also we assume independent lives in the population, we could consider that Q is mean of q and the population consists of $\frac{1+q}{q}$ lives. The population size decreases in term of age as the mortality rate increases and it converges to two, strengthen the facts that we consider greater variability for older ages and we put very low trust on our data.

Since statistical independence is assumed for every pair of abridged mortality rates, calculation for full data sampling model can be simplified by just multiplying sampling model of each data. Our posterior is obtained by multiplying prior densities and sampling model.

$$\text{Posterior} \propto \exp\left(-\frac{\alpha}{0.02347}-\frac{b}{0.00627}-\frac{\varsigma}{0.00189}\right)\times \varsigma^{51.8838}\prod_{i=1}^{11}\left(\frac{\Gamma\left(\frac{1}{q_{O,i}}\right)}{\Gamma\left(\frac{1-q_{O,i}}{q_{O,i}}\right)}\left(q_{O,i}\right)^{\frac{1}{q_{O,i}}-1}\left(1-q_{O,i}\right)^{\frac{1}{q_{O,i}}-2}\right) \quad (8)$$

Since every element of Q_O should be five-year mortality rates and we have changed our parameterisation for Makeham model, now they should be calculated as:

$${}_5q_x = 1-\exp\left(-5(\alpha-B)-\frac{B(\varsigma+1)^x((\varsigma+1)^5-1)}{\log(\varsigma+1)}\right) \quad (9)$$

The likelihood in (8) is also tedious, so we could not do analytical method to get the posterior distribution of the parameters. It is also hard to find full conditional posterior distribution for each parameter, so we could not use Monte-Carlo simulation with Gibbs sampling algorithm. In this case, we would implement Metropolis-Hastings algorithm.

Because it is hard to obtain posterior distribution of q_x, we would estimate expected value of q_x by substituting the expected value of α, B, and Ϛ into:

$$q_x = 1-\exp\left(-(\alpha-B)-\frac{B\varsigma(\varsigma+1)^x}{\log(\varsigma+1)}\right) \quad (10)$$

Using calculus, (the proof is omitted) we can see that q_x is an increasing function of α, B, and Ϛ. Hence, we could define best-estimate of q_x by substituting 2.5-percentile of α, B, and Ϛ into (10), also worst-estimate of q_x by substituting 97.5-percentile of α, B, and Ϛ into (10).

D. Investment Return Estimation

According to International Monetary Fund [52], it is hard to identify the fluctuation trend of inflation, especially for countries with transitioning economics. Seasonal trends that appeared graphically can be occurred by natural variation (e.g. weather) and artificial variation (e.g. price setting by government). In practical analysis, seasonal trend is generally ignored to diminish inaccuracy and confusing situation in interpreting the fluctuation. Appropriate procedure to consider seasonal trend also needs exact knowledge of product availability time after time in the market to make proper adjustments. However, only analyzing annual inflation is also unwise because we might understate the inflation rate itself in the medium term.

In this research, we assumed that monthly investment return can be predicted from monthly inflation rate. In order to keep or increase real purchasing power, investment return must keep up with inflation rate. Inflation is widely calculated by calculating the movement in price index. We had another assumption that monthly price ratios follow a lognormal stochastic process. The process could be used for estimating investment return since the Indonesian investment market follows many assumptions as described by Dickson et al [47]. We have both stocks and zero-coupon bonds in our market, paid dividends can be easily reinvested in stocks, capital market trades are running continuously, transaction fees are low, and Indonesian law of capital market law still permits short selling practices.

To ensure that we can use the lognormal stochastic process assumption, we observed monthly ratio time-series, yearly ratio time-series, and empirical density plot of monthly ratios. No linear trend was observed, implying that the movement is clearly variative and condition of required data for this study was satisfied. Further consideration by running Augmented Dickey-Fuller test on R by using Tseries package [53] supports the hypothesis of time series stationarity for every α ≥ 0.01. Therefore, we could proceed with this data.

Some readers may ask why we denied suggested significance of lag-1 autocorrelation as value of r_1 is around 0.4. First, we are not conducting any forecast of future monthly ratios. We are only interested in understanding about distribution of the monthly ratios and it does not depend on time since the time-series is already stationary. From the distribution, we are looking for estimating the best and worst economic condition possible. Second, it falls in line with obtained data and reality in life that past conditions have a weak effect on future conditions. We tried to examine possibilities of needing more sophisticated ARMA(p, q) models by looking at extended autocorrelation function matrix calculated by TSA package [54]. After fitting the parameters into suggested model specifications, we found that the best model by Akaike Information Criterion is ARMA(2, 3) with AIC value equals -1356.400. When we calculated the R^2 statistics for this 5-parameters model, we only got the value of 0.396 and [55] considered it as a weak effect size.

The lognormal stochastic process assumption means that the logarithm of monthly price ratios follows a normal distribution. Thus, we could use a normal model with unknown population mean and variance as described by [43] (please refer to Section 5.3 for the details). Since we did not have any prior research on this topic, we would construct our prior distribution by averaging results from frequentist fitting methods, including moment matching, maximum likelihood estimation, and percentile matching with three choices of interval width (70%, 90%, and 95%). With that model, we can obtain posterior distribution of both μ and σ. Nevertheless, parameters for σ still contain μ. Therefore, we need to approximate their values using Monte-Carlo simulations. Once again, by assuming that investment return is close to the price ratio, we are going to end this subsection with the distribution of accumulation factor, which equals one plus the return (1+i). Accumulation factor follows lognormal distribution with the posterior distributions of $\mu|\sigma^2$ and σ^2 are going to be written as (11) and (12).

$$\{\mu|y_1,\dots,y_{168},\sigma^2\}\sim N\left(0.0047,\frac{\sigma^2}{299.909}\right) \quad (11)$$

$$\{\sigma^2|y_1,\dots,y_{168}\}\sim \text{Inverse}-\text{Gamma}(149.9545,0.0038) \quad (12)$$

E. Fair Annual Pure Premium Range Estimation

We limit our research on calculating fair pure premium range for one-year term life insurance that starts from an integral age. Theoretically, we assume that benefit payment occurs at the moment of insured death or waiting until the end of the year, when practical condition is actually in the middle of the year. Therefore, it is more plausible for insurance companies to have their calculation based on first assumption, which in actuarial practices we denote them as $\bar{A}^{1}_{x:\bar{1}|}$. By assuming constant discount factor during the year, and knowing the density function of the mortality model, we can calculate the annual pure premium as:

$$\bar{A}^{1}_{x:\bar{1}|}=\int_0^1 v^t f_x(t)dt \quad (13)$$

For young ages, we only have function of annual mortality rates as written in (2). For old ages, we have the Makeham model with a well-defined continuous density function, but integral involving it is very hard to be done analytically. Hence, rather than using (13) for calculation, we approximate its value by:

$$\bar{A}^{1}_{x:\bar{1}|}\approx v^{T_x}q_x \quad (14)$$

In Section II.D, we obtained the distribution of accumulation factor, but we have not obtained distribution of discount factor (v). Since the accumulation factor time series is assumed to follow a stochastic lognormal process with parameters μ and σ, or in another word's accumulation factor is lognormal distributed with parameters μ and σ, we can also assume that the discount factor is having the same distribution with parameters $-\mu$ and σ.

Practically, the premium must be a constant that is fixed for all individuals in a certain age, but our formulation in (14) still treats it as a random variable. Therefore, we estimated the expected value of pure premium by substituting the expected value of q_x, μ, σ, and $e_{x:\bar{1}|}$ (equal to $1-q_x$) as approximation for the expected value of T_x (time of death given death occurred in that year). For the best estimate, we substituted the best estimate of q_x, 97.5-percentile of both μ and σ and t_x to minimize 2.5-percentile value of discount factor distribution with respective μ and σ. For the worst estimate, we substituted the worst estimate of q_x, 2.5-percentile of μ, 97.5-percentile of σ, and t_x to maximize 97.5-percentile value of discount factor distribution with respective μ and σ. In order to simplify the assumptions, we limit t_x value corresponding with the number of days, which means that death in the morning is considered equal as a death in the night, of the same day.

Readers may wonder why we do not simply implement uniform death distribution (UDD) for fractional ages to obtain our expected value of the pure premium. By using UDD, we set $t_x = 0.5$ for all integral ages. That means if someone aged x is going to die in one year, he/she is expected to die in half a year. We consider that this is not appropriate because force of mortality must be increasing in term of age. For younger ages, tendency and probability of dying in the short term is smaller than the older ages. Moreover, old-aged population is more exposed to have sudden death because of aging.

Our results can be extended for stochastic calculation of longer-term insurance products, but it is not covered in this article. Current digital life insurers still focus on a one-year term, when economic difficulties because of the COVID-19 pandemic imply more people are interested in having less long-term products.

III. Results and Discussions

This section explains how we check that our models have obtained convergence and given acceptable accuracy. We also provided our results and how we interpret them for the sake of the life insurance business.

A. Fitting Procedure and Convergence Test Methods

Dellaportas et al. [13] used 100,000 iterations for burn-in and chain thinning for every fifty iterations, with a total of 2500 values to be kept from the full Heligman-Pollard model. Considering their research, we also sampled parameter values by 225,000 iterations for our truncated Heligman-Pollard and Makeham models, each for males and females. Later, we decided not to thin our chains because of inefficiencies [56], [57].

For every step, we set three chains to ensure that posterior estimates do not depend on starting values. Convergence of the chains considered by observing results of time-series trace plots, Heidelberger-Welch stationarity test with critical p-value = 0.01, halfwidth test with $\epsilon = 0.25$, Geweke stationarity test with critical p-value = 0.01, and Geweke-Rubin multichain convergence test.

B. Posterior Distribution of Mortality Model Parameters

Young-aged males' chains converged after 10,000 iterations. We plot their density functions for the sampled iterations in Fig. 3, and it seems that all the parameters are positively skewed. The positively skewed distribution implies that the modes are lower than the means. The accuracy of abridged mortality rates prediction is good [58], with the mean absolute percentage error (MAPE) of 12.74% and Pearson correlation coefficient (r) of 98.54%. Since real values in the data are between their best and worst estimates, we believe that our model does not rely heavily on the data.

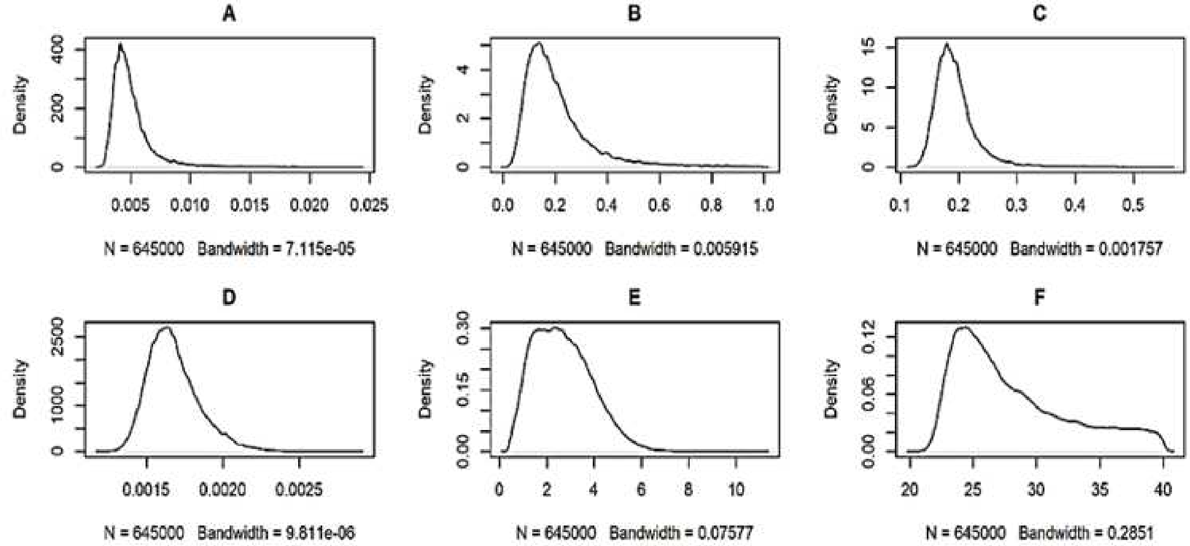

Fig. 3 Estimated posterior distributions for parameters (A and B for babies; C for childhood; D, E, and F for adolescence).

Old aged males' chains converged after 15,000 iterations. For the sampled iterations, we plot their density functions in Fig. 4. According to the plots, distribution of A is close to the exponential distribution, while distribution for other parameters are quite symmetric.

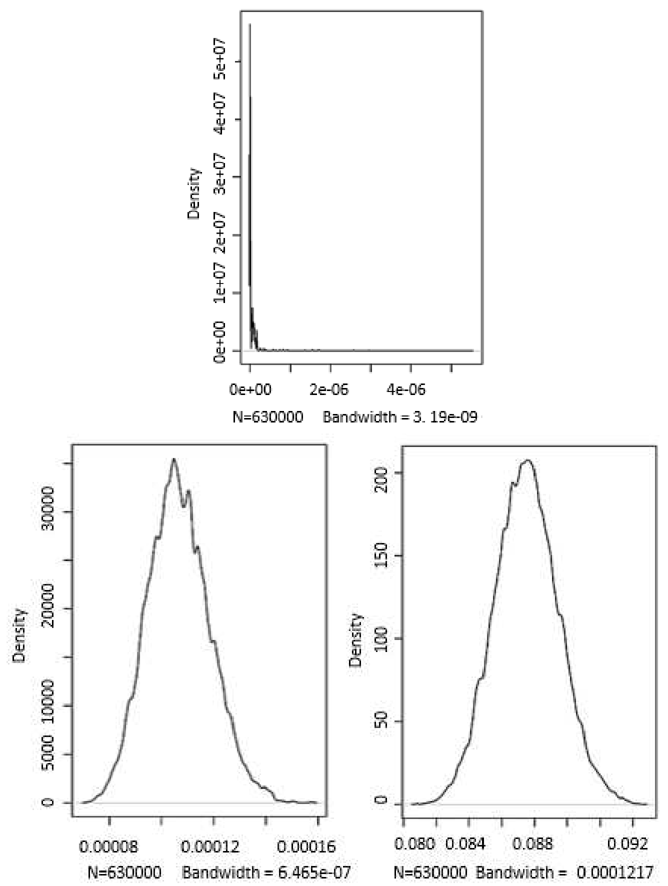

Fig. 4 Estimated posterior distributions for parameters -A (top), B (bottom left) and C (bottom right)- of old-aged males modified Makeham model

Accuracy of abridged mortality rates prediction in fitting data is considerably very accurate with MAPE of 8.66% and correlation of 99.97% with real values in the data are between their best and worst estimates. Extrapolation accuracy is also considered very accurate, with MAPE of 8.92% and correlation of 99.69% with real values in the data being between their best and worst estimates.

Young age females' chains also obtain convergence after 85,000 iterations. Their density functions show that all the parameters are positively skewed, except for parameter F. Accuracy of abridged mortality rates prediction is considerably good with MAPE of 11.68% and correlation of 98.23%. Range of best and worst estimates also contain real values in the data.

Last, 10,000 iterations are needed to reach convergence in old age females' chains. Their density functions are having a similar shape to the respective model for males. Distribution of α is close to the exponential distribution and the rest are quite symmetric. Since MAPE is 9.40% and the correlation is 99.49%, abridged mortality rates prediction in fitting data is considered very accurate.

Real values in the data are also between their best and worst estimates. Extrapolation accuracy of mortality rates is also considered quite good, with MAPE of 19.62% and r correlation of 99.61% with real values in the data are between their best and worst estimates.

C. *Estimated Investment Return*

With previous mortality models, we could proceed to obtain annual mortality rates. We further proceed to the discount factor to consider reasonable investment return assumption during product development, pricing, valuation, and advertising practices. Generally, it seems feasible to assume that the investment return can be maximized for a long time of investment, or large loss was incurred when we enter the market in a bad condition and have to get out from the market soon. It falls in line with our estimate. The best case for insurance companies occurs if deaths happen in the end of the year (especially if the economic condition is at its best, see Fig. 5).

On the other side, worst case occurs if deaths happen in the first day of protection (especially if the economic condition is at its worst). It is also estimated that insurance companies are guaranteed to be safe from investment losses only if deaths do not occur in first 58 days of protection. By our calculation, the companies should anticipate their return to fall in the range of (-5.68%, 6.81%). Since the upper limit is not so high, we recommend that insurance companies with unexperienced investment managers to put most of their money in safer investment instruments: banking deposits, government bonds, and high-quality corporate bonds.

Fig. 5 Discount factor in the best economic condition.

D. Estimated Investment Return

Estimated annual pure premium range for males (M) and females (F) at some ages are provided in Table II, specifically for age last birthday (denoted as x). Those rates are applicable for sum assured worth one million.

E. Discussions

Based on our results, we have calculated 100-years life expectancy since newborn ($e_{0:\overline{100}|}$) for both males and females. For males, the best estimate is 77.51 years, the expected value is 69.74 years, and the worst estimate is 55.46 years. For females, the best estimate is 81.07 years, the expected value is 73.83 years, and the worst estimate is 55.69 years. Those can be alert for Indonesian to have put high concern on healthy lifestyle since worst estimate for both males and females are under 56 years old, which is set as pension age by Indonesian government for civil servants [59].

TABLE II
ESTIMATED ANNUAL PURE PREMIUM RANGE

x	Best estimate		Expected value		Worst estimate	
	M	F	M	F	M	F
0	4436	3124	19057	15516	129564	158641
1	1233	444	3928	4073	10322	14831
5	60	4	644	452	3585	5196
10	8	7	322	319	3070	3548
15	13	77	671	550	3123	2982
20	129	256	1235	785	3063	2661
25	473	489	1575	945	2902	2426
30	841	636	1202	899	1873	1393
40	1976	1486	2912	2162	4684	3456
50	4506	3370	6857	5058	11396	8347
60	10138	7536	15928	11676	27329	19886
70	22601	16715	36619	26717	64580	46821
80	49851	36761	82933	60439	148576	108107

We have also calculated the probability of a newborn to survive until 100 years old (${}_{100}p_0$). For males, the best estimate is 3.83%, the expected value is 0.36%, and the worst estimate is 0.06‰. For females, the best estimate is 9.28%, the expected value is 1.78%, and the worst estimate is 0.93‰. Since the best estimate is still over 1% for both males and females, importance of preparedness to face longevity risk is still significant.

Assessing the significance of accident hump, previously we defined young-aged population as individuals aged 30 years old or below, so we expected that accident hump happens before age of 30 if it occurs. Posterior mode of parameter F satisfies that empirical inspection of accident hump in age interval (20, 25) exists for males. Even the posterior mean and median are higher than 25, they still fall below 30. Posterior probability estimate of F is higher than 30 equals 28.3%, which means that the possibility of having the accident hump is higher than not for males.

However, empirical accident hump for females in age interval (15, 20) was not fulfilled by the parameter E of the posterior distribution, even the numerical posterior probability estimate of F in that range is zero. Further checking as to why it happens is required, it might imply the lack of our approach in detecting the hump. Other possible explanation is that there is no accident hump in Indonesian female population since observed posterior mean, median, and mode of F are all higher than 30, also posterior probability estimate of F is higher than 30 equals 84.8%. Further examination at Table II gives us another insight that we must be cautious to set premium tariff for males and females. Even though, generally females have lower mortality risk for a certain age, sometimes they face the opposite. Females have higher best estimate of q_x for X = {11, 12, …, 24, 25}, higher expectation of q_x for X = {1, 11}, and higher worst estimate of q_x for X = {0, 1, …, 12, 13}.

The interval estimates of pure premium are provided since we were not fully certain about the true pattern of mortality and investment return in population. This research is based on assumption that need to be further examined, thus, a more flexible result in the form of interval estimation is preferred than the mere point estimation, to allow for some variations to the real condition.

We also recommended more anticipation of negative return since economic growth is slowing nowadays and we have to anticipate the worst condition. Especially, when this research was conducted, the world is facing coronavirus with huge effect in economics.

Beside of our results, internal experience study is still needed for insurance companies to determine their quoted premium for the customers. However, new companies without any self-owned data might consider numbers between our expected value and worst estimate in good economic conditions. Otherwise, numbers between our best estimate and expected value can be considered during economic recession and depression to make sure that customers can afford the products.

IV. CONCLUSION

This article proposed implementation of Bayesian method with beta sampling model and Metropolis-Hastings algorithm to estimate Indonesian complete life table from its abridged life table. We assumed compliance of truncated Heligman-Pollard and Makeham mortality models to construct underlying pattern of yearly mortality rates. We also estimated reasonable annual investment return range by Bayesian normal model and Monte-Carlo simulation method. Estimates from the two independent processes were combined to construct fair annual pure premium range for 0 – 99 years old Indonesian population.

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