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Analysis of Waiting Time Using Three Mixtures Exponential Distribution

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***Abstract*—Mixed exponential distributions play a significant role in lifetime data analysis, but if we use traditional statistical methods to estimate the parameters in the model, it will be very difficult. However, we employed the Expectation-Maximization (EM) algorithm to estimate the parameters of the model. It will simplify the complexity of the calculation. This paper studies the parameter estimation problem in a complete data situation and gives Monte Carlo (MC) simulation. The EM algorithm is good to estimate the parameters for the three mixed exponential distributions. The parameters estimating were remarkably close to the real values; simultaneously, the samples' RMSE values are more and smaller along with the increase of the sample size so that the method can be regarded as a kind of very effective statistical analysis calculation method. Results show that the algorithm based on EM to estimate the parameters of the mixed exponential distribution is remarkably effective. An application was made at the three phases waiting time in the Rasheed Bank in AL-Mustansiriyah University. The results showed that the estimating mean waiting time by EM algorithm for then the audit stage phase has the biggest proportional in this process which has formed (48%) from total mixture distribution component with scale parameter (0.46 hours), then the provide information phase (32%) with scale parameter (0.44 hours), then the stage of the cashier (20%) with mean waiting time (0.32 hours).**

***Keywords*—EM algorithm; RMSE; Monte Carlo (MC); mixtures exponential; Rasheed Bank.**

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I. INTRODUCTION

In engineering, medicine, biology, etc., failure and lifetime data analysis have become a problem that statisticians are concerned about. There are many statistical methods about failure and life data analysis for a single population, but there often is more than one population in the practical. Therefore, the study of the mixed parameter estimation will become especially important. Gallaugher and McNicholas [1] give parameters estimation of the single mixed exponential distribution with EM algorithm. Sarabia *et al.* [2] put forward the EM algorithm, which simplifies the calculation of maximum likelihood estimation, but when there is no explicit format in maximizing is developed quickly and is widely used and gives out the generalized EM algorithm (GEM) [3], [4]. This paper gives the estimation of the parameters of three mixed exponential distributions with the EM algorithm provides the MC simulation, furthermore applied EM algorithm to studying and analyzing the three phases of waiting time for AL-Rasheed Bank in AL-Mustansiriyah University clients.

II. MATERIALS AND METHOD

A. The Mixture Exponential Distribution

Consider the following five parameters probability density for the subpopulations $Z \sim Exp(\alpha_j)$, $j = 1,2,3$:

$$f(z_i, \alpha_j) = \alpha \, e^{-\alpha \, z_i} \quad , \; z_i, \alpha_j > 0 \;, \; j = 1,2,3 \qquad (1)$$

and the population Z satisfies:
$P(Z = z_1) = \theta_1$, $P(Z = z_2) = \theta_2$, $P(Z = z_3) = \theta_3$, $\theta_3 = (1 - \theta_1 - \theta_2)$

Then this population back to the three mixed exponential distributions, which probability density function [5] is as follows:

$$f(z, \alpha_j, \theta_j) = \theta_1 f_1(z, \alpha_1) + \theta_2 f_2(z, \alpha_2) + (1 - \theta_1 - \theta_2) f_3(z, \alpha_3) \quad (2)$$

where:

$f_{1i}(z_i,\alpha_1) = \alpha_1\, e^{-\alpha_1 z_i}$, $f_{2i}(z_i,\alpha_2) = \alpha_2\, e^{-\alpha_2 z_i}$,
$f_{3i}(z_i,\alpha_3) = \alpha_3\, e^{-\alpha_3 z_i}$

$z_i > 0$, $0 < \alpha_j < 1$, $j = 1,2,3$

θ_j represent the proportional parameters for each subpopulation z_j, such as
$\theta_j > 0$, $j = 1,2,3$.
If $Z_1, Z_2, \dots, Z_n$ are a random sample for the mixed exponential distributions, and $z_1, z_2, \dots, z_n$ are the observed values for the samples, then:
$f_{1i}(z_i,\alpha_1) = \alpha_1\, e^{-\alpha_1 z_i}$, $f_{2i}(z_i,\alpha_2) = \alpha_2\, e^{-\alpha_2 z_i}$,
$f_{3i}(z_i,\alpha_3) = \alpha_3\, e^{-\alpha_3 z_i}$

$$f_i(z_i,\alpha_j,\theta_j) = \theta_1 f_{1i}(z_i,\alpha_1) + \theta_2 f_{2i}(z_i,\alpha_2) \qquad (3)$$

For a random variable v, which satisfies [6]:
$P(v=1) = \theta_1$, $P(v=2) = \theta_2$, $P(v=3) = \theta_3$
Then z_i submits to the three mixed exponential distributions z, so the joint distribution between z and v is given by:

$$f_i(z_i,v_j,\alpha_j,\theta_j) = \begin{cases} \theta_1 f_{1i}(z_i,\alpha_1) & when\ v = 1 \\ \theta_2 f_{2i}(z_i,\alpha_2) & when\ v = 2 \\ \theta_3 f_{3i}(z_i,\alpha_3) & when\ v = 3 \end{cases} \qquad (4)$$

Then for given z_i, the marginal distributions for v are given by:

$$P(v=1|z_i,\alpha_j,\theta_j) = \frac{\theta_1 f_{1i}(z_i,\alpha_1)}{f_i(z_i,\alpha_j,\theta_j)} \qquad (5)$$

$$P(v=2|z_i,\alpha_j,\theta_j) = \frac{\theta_2 f_{2i}(z_i,\alpha_2)}{f_i(z_i,\alpha_j,\theta_j)} \qquad (6)$$

$$P(v=3|z_i,\alpha_j,\theta_j) = \frac{\theta_3 f_{3i}(z_i,\alpha_3)}{f_i(z_i,\alpha_j,\theta_j)} \qquad (7)$$

Figure (1) represents the three-mixture exponential distribution. It is shown that the Exponential mixture distribution is right-skewed. This skew is decreased as θ increases [7]. Also, the crest of the distribution becomes less sharp when its tail becomes shorter as α increases. This means that the kurtosis of the mixture distribution decreases as α increases [8].

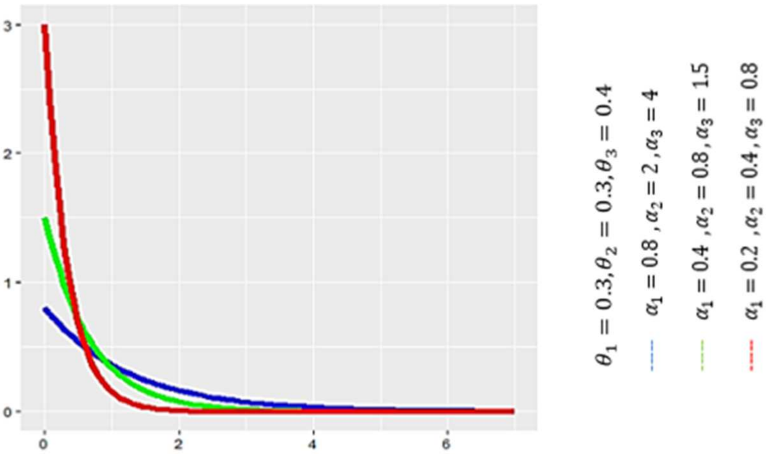

Fig.1 Density of mixture exponential distribution

B. EM algorithm Parameter Estimation

One can estimate the mixture exponential distribution parameters depend on the EM algorithm, for a given the initial parameter values $\tau_o = (\theta_{1o},\theta_{2o},\theta_{3o},\alpha_{1o},\alpha_{2o},\alpha_{3o})$, thus we can follows[9], [10]:

C. M-Step: Expectation for m=1,2,…

$$\varphi(\tau|\tau_{m-1}) = \sum_{i=1}^{n} E_v\,\{ln\, f\,(z_i, v|\tau_o,\tau_{m-1})\}$$

$$\varphi(\tau|\tau_{m-1}) = \sum_{i=1}^{n} \left\{ \frac{\theta_{1(m-1)} f_{(m-1)1i}}{f_{(m-1)i}}\ ln(\theta_1\, f_{1i}) + \frac{\theta_{2(m-1)} f_{(m-1)2i}}{f_{(m-1)i}}\ ln(\theta_2\, f_{2i}) + \frac{\theta_{3(m-1)} f_{(m-1)3i}}{f_{(m-1)i}}\ ln(\theta_3\, f_{3i}) \right\} \qquad (8)$$

$$\varphi(\tau|\tau_{m-1}) = \sum_{i=1}^{n} \{G_{(m-1)1i} ln(\theta_1\, f_{1i}) + G_{(m-1)2i} ln(\theta_2\, f_{2i}) + G_{(m-1)3i} ln(\theta_3\, f_{3i})\}$$

$$(\tau|\tau_{m-1}) = \sum_{i=1}^{n}[G_{(m-1)1i}\{ln(\theta_1) + ln(\alpha_1) - \alpha_1 z_i\} + G_{(m-1)2i}\{ln(\theta_2) + ln(\alpha_2) - \alpha_2 z_i\} + G_{(m-1)3i}\{ln(\theta_3) + ln(\alpha_3) - \alpha_3 z_i\}] \qquad (9)$$

Where:
$\tau_{m-1}: \left(\theta_{1(m-1)}, \theta_{2(m-1)}, \theta_{3(m-1)}, \alpha_{1(m-1)}, \alpha_{2(m-1)}, \alpha_{3(m-1)}\right)$
$f_{(m-1)i}: f_i(z_i,v_j,\alpha_j,\theta_j)$
$f_{(m-1)1i}: \theta_1 f_{1i}(z_i,\alpha_1)$, $f_{(m-1)2i}: \theta_2 f_{2i}(z_i,\alpha_2)$,
$f_{(m-1)3i}: \theta_3 f_{3i}(z_i,\alpha_3)$
$G_{(m-1)1i}: \dfrac{\theta_{1(m-1)} f_{(m-1)1i}}{f_{(m-1)i}}$,
$G_{(m-1)2i}: \dfrac{\theta_{2(m-1)} f_{(m-1)2i}}{f_{(m-1)i}}$,
$G_{(m-1)3i}: \dfrac{\theta_{3(m-1)} f_{(m-1)3i}}{f_{(m-1)i}}$

D. M-Step for τ_m

In this step, we select the values of τ that maximization (9), such as:
$\varphi(\tau|\tau_{m-1}) = max\{\varphi_\tau(\tau|\tau_{m-1})\}$

To get the EM estimator (optimal estimate) for τ, if τ_m represent a new initial value of the parameters τ, repeat steps (1) and (2) above until we get $\|\tau_m - \tau_{m-1}\| < \sigma$, where σ is the given threshold value in advance, then stop the iteration [11].

Now because $\varphi(\tau|\tau_{m-1})$ is a transcendental equation about τ: $(\theta_1, \theta_2, \theta_3, \alpha_1, \alpha_2, \alpha_3)$, it is difficult to solve the equation directly $\frac{\partial\varphi(\tau|\tau_{m-1})}{\partial\tau} = 0$, thus we can estimate the parameter τ by using Newton iterative method as the following steps [12]:

- Set $\tau = (\tau_1, \tau_2, \dots, \tau_k)$, the above step (2) is decomposed by the following conditional of maximization for (k) times.
- Let $\tau_{(m-1)} = \{\tau_{(m-1)1}, \tau_{(m-1)2}, \dots, \tau_{(m-1)k}\}$, then in the M-Step iteration, let $\tau_2 = \tau_{(m-1)2}, \dots, \tau_k = \tau_{(m-1)k}$, and solve $\varphi(\tau_{m1}|\tau_{m-1}) = max\{\varphi_{\tau 1}(\tau|\tau_{m-1})\}$
- Let $\tau_1 = \tau_{(m-1)1}, \tau_3 = \tau_{(m-1)3}, \dots, \alpha_k = \alpha_{(m-1)k}$ and solve $\varphi(\tau_{m2}|\tau_{m-1}) = max\{\varphi_{\tau 2}(\tau|\tau_{m-1})\}$.
- Repeat this approach (k) times, then complete iteration to get $\tau_m = \tau_{m1}, \tau_{m2}, \dots, \tau_{mk}$.

Then when $k = 5$ we can get the EM parameters estimation as following [13]:

- Estimation of $\theta_1, \theta_2, \theta_3$

$$\frac{\partial\varphi(\tau|\tau_{m-1})}{\partial\theta_1} = \sum_{i=1}^{n}\left\{\frac{G_{(m-1)1i}}{\theta_1} - \frac{G_{(m-1)3i}}{\theta_3}\right\}$$

$$\frac{\partial\varphi(\tau|\tau_{m-1})}{\partial\theta_1} = \sum_{i=1}^{n}\left\{\frac{\theta_3\, G_{(m-1)1i} - \theta_1 G_{(m-1)3i}}{\theta_1\, \theta_3}\right\} \tag{10}$$

and

$$\frac{\partial\varphi(\tau|\tau_{m-1})}{\partial\theta_2} = \sum_{i=1}^{n}\left\{\frac{G_{(m-1)2i}}{\theta_2} - \frac{G_{(m-1)3i}}{\theta_3}\right\}$$

$$\frac{\partial\varphi(\tau|\tau_{m-1})}{\partial\theta_2} = \sum_{i=1}^{n}\left\{\frac{\theta_3\, G_{(m-1)2i} - \theta_2 G_{(m-1)3i}}{\theta_1\, \theta_3}\right\} \tag{11}$$

Putting $\frac{\partial\varphi(\tau|\tau_{m-1})}{\partial\theta_1} = 0$, $\frac{\partial\varphi(\tau|\tau_{m-1})}{\partial\theta_2} = 0$, then the solutions are:

$$\theta_1 = \frac{1}{n}\sum_{i=1}^{n} G_{(m-1)1i} \;,\; \theta_2 = \frac{1}{n}\sum_{i=1}^{n} G_{(m-1)2i} \;,\; \theta_3 = 1 - \theta_1 - \theta_2 \tag{12}$$

- **Estimation of $\boldsymbol{\alpha_1, \alpha_2, \alpha_3}$**

$$\frac{\partial\varphi(\tau|\tau_{m-1})}{\partial\alpha_j} = \sum_{i=1}^{n}\left\{G_{(m-1)ji}\left(\frac{1}{\alpha_j} - z_i\right)\right\}$$

By letting $\frac{\partial\varphi(\tau|\tau_{m-1})}{\partial\alpha_j} = 0$, we get:

$$\alpha_{mj} = \frac{\sum_{i=1}^{n} G_{(m-1)ji}}{\sum_{i=1}^{n} G_{(m-1)ji}\, z_i} \tag{13}$$

III. Results and Discussion

In the simulation study, we use R programming to simulate three different samples from an exponential distribution, these samples with sizes (n=25,50,100), the simulation proportion and scale parameters is designed according to the following Table (1).

TABLE I
SIMULATION FOR (PROPORTION AND SCALE) PARAMETERS

Model	**Real Scale Parameters**	**Initial Scale Parameters**	**Initial Proportion Parameters**
1	$\alpha_1 = 0.1, \alpha_2 = 0.5, \alpha_3 = 1$	$\alpha_1 = 0.05, \alpha_2 = 0.4, \alpha_3 = 0.8$	$\theta_1 = 0.3, \theta_2 = 0.3, \theta_3 = 0.3$
2	$\alpha_1 = 0.5, \alpha_2 = 1, \alpha_3 = 2$	$\alpha_1 = 0.3, \alpha_2 = 0.75, \alpha_3$	
3	$\alpha_1 = 1, \alpha_2 = 2, \alpha_3 = 3$	$\alpha_1 = 0.5, \alpha_2 = 1, \alpha_3 = 2.5$	

We used formulas 12 and 13 to find EM estimators, (r=1,2, … ,1000) was used as a number of replications. The mean and root mean square error of estimators is calculated as follows [14], [15]:

$$\hat{\tau} = \frac{1}{1000}\sum_{r=1}^{1000}\hat{\tau}_r \quad , \quad RMSE(\hat{\tau}) = \sqrt{\frac{1}{r}\sum_{r=1}^{1000}(\hat{\tau}_r - \tau)^2}$$

The results are summarized and tabulated in Table (2) and Table (3) below. These tables include parameters estimators and mean square error (MSE) for those estimators [16].

TABLE II
EM PROPORTION AND SCALE PARAMETERS ESTIMATORS

model	n	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$
1	25	0.36	0.38	0.26	0.161	0.608	1.187
	50	0.28	0.36	0.36	0.156	0.582	1.119
	100	0.31	0.32	0.37	0.153	0.563	1.081
2	25	0.20	0.19	0.61	0.615	1.191	2.297
	50	0.38	0.39	0.23	0.581	1.105	2.189
	100	0.27	0.29	0.44	0.566	1.085	2.123
3	25	0.17	0.16	0.67	1.136	1.763	3.775
	50	0.19	0.21	0.60	1.091	1.603	3.31
	100	0.26	0.24	0.50	1.067	1.634	3.208

TABLE III
RMSE FOR EM PROPORTION AND SCALE PARAMETERS ESTIMATORS

model	n	$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$
1	25	0.061	0.085	0.141	0.061	0.108	0.187
	50	0.023	0.064	0.043	0.056	0.082	0.119
	100	0.016	0.022	0.038	0.053	0.063	0.081
2	25	0.117	0.116	0.216	0.115	0.191	0.297
	50	0.085	0.092	0.177	0.081	0.105	0.189
	100	0.033	0.011	0.041	0.066	0.085	0.123
3	25	0.136	0.144	0.272	0.136	0.237	0.775
	50	0.112	0.092	0.235	0.091	0.396	0.31
	100	0.044	0.063	0.101	0.067	0.337	0.208

It is obvious from Tables (2) and (3) above:

- The estimated values of the scale parameters are close to the real values as the sample size increase.
- When the scale parameters increase, the estimated parameters will pull away from the real values [17].
- In the EM estimated for the proportional parameters, the estimated values for these parameters under the three mixture models recorded the biggest proportional parameter for the first component, then followed by the second component, then the third one.
- The (RMSE) decreases as sample size increases, while it is increased when scale parameters increase.

A. Application

The theoretical study of this research has been applied in studying and analyzing the waiting times (in hours) that customers spent who have current accounts with Al-Rasheed Bank/Mustansiriyah University. This process passes through three phases. Provide information on the withdrawal process, then the audit stage, and then the receipt of the amounts of those instruments from the cashier. Table (4) shows the waiting times for the bank's customers recorded for a sample included (25) clients.

TABLE IV
WAITING TIMES FOR THE AL-RASHEED BANK 'S CUSTOMERS

Costumer	**Phase**										
	I	**II**	**III**		**I**	**II**	**III**		**I**	**II**	**III**
1	0.41	0.08	0.02	11	0.01	0.15	0.47	21	0.01	0.98	0.18
2	0.11	0.34	1.68	12	0.69	0.51	0.42	22	1.47	0.12	1.13
3	0.16	0.21	0.4	13	0.83	0.22	0.65	23	0.58	0.86	0.14
4	0.81	0.09	0.34	14	0.46	0.3	0.07	24	0.18	0.17	0.2
5	0.63	0.52	0.91	15	0.81	0.14	0.68	25	0.47	0.04	0.57
6	0.15	0.02	0.04	16	0.12	0.06	0.07				
7	0.48	0.07	0.12	17	0.82	0.12	0.02				
8	0.18	0.23	0.09	18	0.39	0.09	0.26				
9	0.73	1.58	0.58	19	0.17	0.36	0.01				
10	1.12	0.01	0.38	20	0.19	0.55	0.03				

We used both Kolmogorov-Smirnov and Anderson-Darling tests to determine whether the above data follows an exponential distribution, according to the following hypothesis under (0.05) significance level [16].
H_0: Waiting time have followed Exponential distribution.
H_1: Waiting time have not followed Exponential distribution.

TABLE V
ANDERSON-DARLING TEST

Phase	K-S Statistic	P-Value	A-D Statistic	P-Value
I	0.553	0.831	0.426	0.611
II	0.663	0.752	0.325	0.855

Clearly, from Table (5), all P-Values are greater than the significance level, which means the waiting time's data follows Exponential distribution. Now we will find the EM's estimators for the $\tau = (\tau_1, \tau_2, \tau_3)$ by using (R) programming, table (6) shown these results.

TABLE VI
PARAMETER ESTIMATION FOR THE REAL DATA

$\hat{\theta}_1$	$\hat{\theta}_2$	$\hat{\theta}_3$	$\hat{\alpha}_1$	$\hat{\alpha}_2$	$\hat{\alpha}_3$
0.48	0.32	0.2	0.44	0.46	0.32

From the results in Table (6) above, which gives estimating parameters of the mixed exponential distribution by the EM method, we can say that the best mixing ratio for the three waiting phases was (48%), (32%), and (20%), respectively, and the mean waiting times for each phase were (0.44), (0.46) and (0.32) hours. Figure (2) below represents the estimated density of the three-mixture exponential distribution waiting time in the bank. Clearly that the mixed phases banks waiting time have right skewed. This skew is decreased as θ increases.

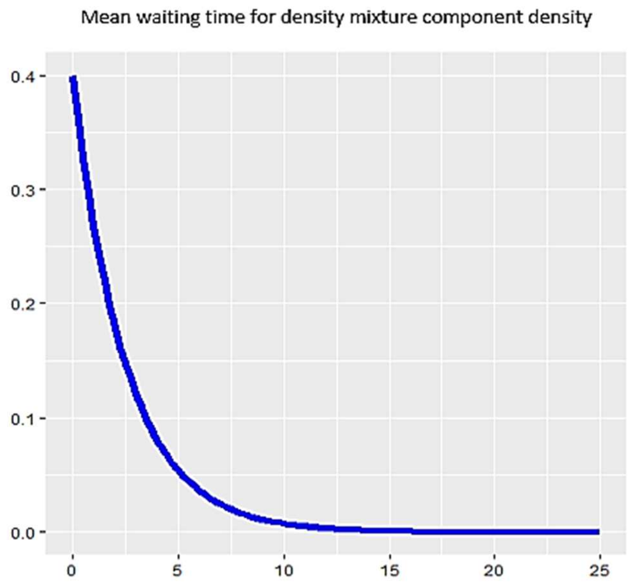

Fig.2 Density of mixture three phases waiting times.

IV. CONCLUSION

The three mixture exponential distribution right-skewed distribution, this skew is decreased as the scales parameters θ s increases. The proportional parameters are very important in the estimation of this model, such as the larger value of the scale α gives the bigger meantime, this leads decreases the model skewness. The EM algorithm is good to estimate the parameters for the three mixed exponential distributions. The parameters estimating were very close to the real values; simultaneously, the samples' RMSE values are more and smaller along with the increase of the sample size, so the method can be regarded as a kind of very effective statistical analysis calculation method. The time spent by the customers of Al Rasheed Bank/Mustansiriyah University can be reduced by reducing administrative circles to develop the bank's work and facilitate the provision of banking services to customers.

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