International Journal on Advanced Science Engineering Information Technology

Vol.10 (2020) No. 3
ISSN: 2088-5334

Study on Error Correction Capability of Simple Concatenated Polar Codes

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Abstract*—Polar codes are mathematically proven to achieve the Shannon limit, where the error probability is reduced with the help of frozen bits. Since the frozen bits are detrimental in terms of transmission efficiency, this paper investigates the importance of the frozen bits and the possibility of being replaced by other protected bits via a concatenation with other outer channel coding schemes. We evaluate the impact of frozen bits to the capability of error correction of original Polar codes (OPC) and the concatenated Polar codes (CPC) in short block-length in terms of bit-error-rate (BER) performances. Repetition codes are used as outer channel encoder prior to the Polar codes and are divided into two schemes, i.e., (i) irregular repetition-CPC (IR-CPC) codes and (ii) regular repetition-CPC (RR-CPC) codes. We evaluate BER performances using computer simulations based on Log-Likelihood Ratio (LLR) with the modulation of Binary Phase Shift Keying (BPSK) under Additive White Gaussian Noise (AWGN) and frequency-flat Rayleigh Fading channels. We found that the OPC is better than the IR-CPC codes or RR-CPC codes for the same channel coding rate and block-length. This finding indicates that the frozen bits in OPC has strong contribution to the error correction capability of the Polar codes and may not be replaced by other bits even though the bits are protected by other channel coding schemes.

Keywords– BER; BPSK; IR-CPC; LLR; Polar codes, RR-CPC.

I. Introduction

Claude E. Shannon said that the damage of transmission caused since the transmission is vulnerable due to the changes of the channel.by noise or storage medium can be reduced using a channel coding technique [1], if the channel coding rate R is below the channel capacity C. Therefore, development of channel coding scheme is of interest for wireless communications.

The third generation partnership project (3GPP) sets Polar coding scheme as one of the channel coding techniques used in the fifth generation of telecommunications (5G) [2]. Polar codes are coding technique that work effective on channels without memory. Polar codes are mathematically proven to achieve Shannon limits [3] having encoders and decoders with low computational complexity [4] and [5].

Polar codes use the concept of channel polarization [3], where the channels are polarized based on the erasure probability to obtain the polarized channel capacity for the given location of the bits. Channels with high capacity are good channels and used to send the information, while the channels with low capacity are bad channels and used for the frozen bits.

Frozen bits are important bits sent along with information bits, of which the values are known to the receiver. Frozen bits help decoder of Polar codes minimize the error, however, frozen bits are detrimental to the transmission efficiency. Furthermore, the performances of Polar codes are depending on the Bhattacharyya parameters. When the Rayleigh fading channel is considered [6], the Bhattacharyya are varying since they depend on the signal-to-noise ratio (SNR), which is changing in frequency-flat Rayleigh fading channels.

We have developed pattern of frozen location based on

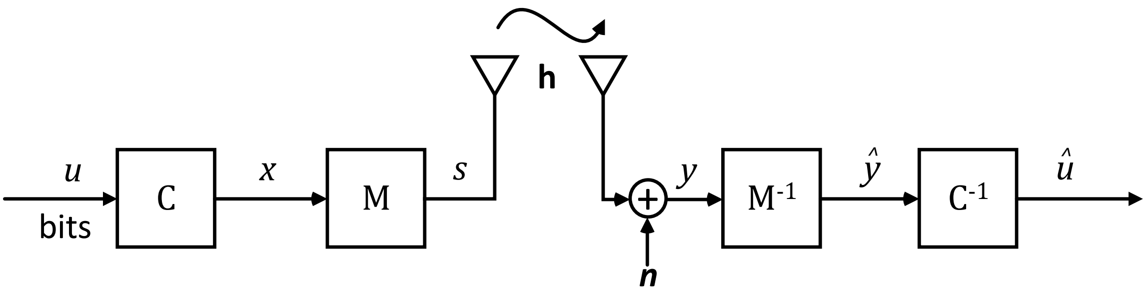

Fig. 1. System model of transmission structure using Repetition-Concatenated Polar codes transmitted under channel h.

Bhattacharyya parameter in [7] resulting good bit-error rate (BER) performances almost for all SNR regime. Since the decoder part leads to a substantial increase in capability of Polar codes in performing error correction, the current development of Polar codes are mostly in the decoder part, for example in [8]–[10].

To investigate the importance of frozen bits, we propose repetition codes to be concatenated with Polar codes such that frozen bits can be protected. Repetition codes, although are very simple, have excellent performances, when they are combined with extended mapping like in [11]. The repetition codes are also providing excellent performances when they are optimized using extrinsic information transfer (EXIT) chart resulting in very close performances to the Shannon limit [12]. Some details of EXIT charts are presented in [13] revealing the benefit, contribution, and connection of frozen bits with the channel capacity.

Repetition codes considered in this paper are with regular repetition and irregular repetition. Ref. [14] proved that combining Polar codes with repetition can increase the security level of Polar codes. This paper presents a different structure of the repetition concatenated polar codes compared to some previous publications, for example, in [14], [15], [16]. Other concatenations are for multiple input multiple output (MIMO) [17] and for source coding [18].

In particular, we investigate how frozen bits are important and how they are likely to be replaced by other coding techniques, whereas [14], [15] combines repetition with Polar codes to improve security and capability without evaluating frozen bits. Other concatenations are discussed in [19].

The contributions of this paper are summarized as follows:

(i) This paper introduces simple concatenation of Polar codes with regular and irregular repetition codes.

(ii) We provide an analysis of error correction capability in terms of BER in AWGN and Rayleigh fading channels, where we found that frozen bits in non-concatenated Polar codes are strong in providing contributions to the error correction capability, which is better than the frozen bits even protected by other channel coding schemes for the same code rate.

The rest of this paper is organized as follows. Section II explains Repetition-concatenated Polar codes system design. Section III evaluates the performances. Section IV concludes the paper.

II. Materials and Method

There is no specific material required to implement either OPC or CPC codes. A series of computer simulations is conducted to evaluate the performance of the OPC and CPC with the designed frozen bits. The system model is illustrated in Fig. 1. We consider u as the information bits sent in the block length of $N = 16$ bits with the channel coding rate $R = 3/16$. Block C is an encoder u to x. In this paper, we use repetition-concatenated Polar codes to evaluate the efficient of frozen bits protected by either regular or irregular repetition codes.

The encoded (x) is modulated by modulator $\mathbf{M}$ using binary phase shift keying (BPSK) modulation. Modulated signal (s) is sent through the channels and received at the receiver as

$$y = h \cdot x + n, \tag{1}$$

with $h = 1$ for the AWGN channel and

$$h = \frac{A + j \cdot B}{\sqrt{2}}, \tag{2}$$

for block Rayleigh fading with $A \sim (\mathcal{N}, 1), B \sim (\mathcal{N}, 1)$ follows the Gaussian distribution with variance of 1 and zero mean, which is in MATLAB written with $randn$ function and $j = \sqrt{-1}$. Variables random $n \sim (\mathcal{N}, \sigma^2)$ is the AWGN noise vector having a Gaussian distribution with variance σ^2 and zero mean. We consider narrowband transmissions. However, the extension to broadband transmission is rather straightforward.

Signal $\mathbf{y}$ is received by the receiver antenna and demodulated as $\hat{y}$. Log-likelihood ratio (LLR) of y is then taken for decoding. The final stage is the process of decoding $\mathbf{C^{-1}}$ with successive cancellation decoding. The results are then used for repetition decoding prior to the conversion to its original bit using hard decision.

The design of IR-CPC and RR-CPC codes is with $R = 3/16$ with a repetition rate to 9 resulting a repetition coding rate of $R = 1/9$. The detailed scenarios for IR-CPC and RR-CPC codes are shown in Table I and Fig. 2.

Fig. 2 shows bipartite graphs of the parity check matrix of IR-CPC and RR-CPC codes with the degree distribution of

$$\Lambda(x) = \frac{1}{16}x^2 + \frac{1}{16}x^3 + \frac{1}{16}x^4. \tag{3}$$

TABLE I
SCENARIO OF REPETITION-CPC CODES AND THE OBTAINED CHANNEL CODING RATE R.

Repetition Concatenated Polar Codes	Rate (R)			Scenario
	Encoding Component			
	Polar codes	Irregular Repetition codes	Regular Repetition codes	
IR-CPC codes	9/16	3/9	-	3/16 (Repetition 9)
RR-CPC codes	9/16	-	3/9	3/16 (Repetition 9)

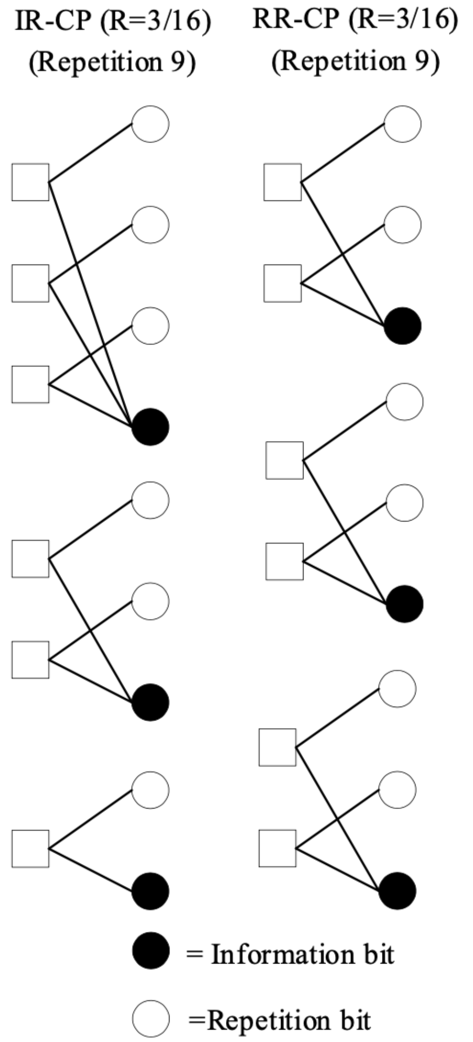

Fig. 2. Bipartite graph for parity check matrix of IR-CPC and RR-CPC codes for coding rate of $R = 3/16$.

and

$$\lambda(x) = \frac{3}{16}x^3. \quad (4)$$

for $R = 3/16$, respectively, where black circle represents the information bits.

A. Transmitter

Based on the system model presented in Fig 1, the transmitter has two main processes, encoder and modulator. Encoder is based on Polar codes, while the modulator is mapping the encoded bits into BPSK symbols. The initial information bit u is encoded with some frozen bits. The frozen bits position is determined using the Bhattacharyya parameters, for binary erasure channel (BEC), as:

$$Z(W) = e^{-R \cdot \frac{E_b}{N_0}} \quad (5)$$
$$Z(W^-) = 2Z(W) - Z(W)^2, \quad (6)$$
$$Z(W^+) = Z(W)^2, \quad (7)$$

which is the opposite of mutual information and capacity taking value between 0 and 1 for BEC and E_b/N_0 is the energy bit per noise spectral density. The channel capacity is therefore $C = 1 - Z(W)$. $Z(W^-)$ and $Z(W^+)$ is the Bhattacharyya for 2×2 kernel Polar codes as in Fig. 3 used to determine the position of information bits and frozen bits, of which the channel is polarized to $C_1 = 1 - Z(W^-)$ and $C_2 = 1 - Z(W^+)$.

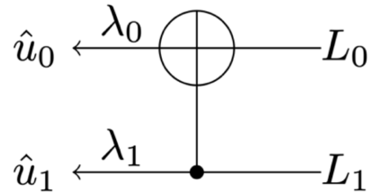

Fig. 3. A 2×2 kernel of Polar codes with channel capacity W.

This paper assumes that u in Fig. 1 has included frozen bits, which by default set to 0 in this paper. The Polar codes encoder can be modelled using matrix multiplication operations between $\mathbf{u}$ with the generator matrix corresponding to the block length codes $N = 2^n$ expressed as

$$G_m = T_2^{\otimes n}, \quad (8)$$

where $\otimes$ is a Kronecker operation. The encoded bits are then expressed as

$$x^N = u^N \cdot G_m. \quad (9)$$

x^N is the output of encoding with blocklength N. In this paper, the decoder uses a simple successive cancellation decoding, of which the advanced version has been presented in [20].

B. Receiver

Based on the CPC transmission system model presented in Fig. 1, the receiver has two main blocks, i.e., demapper

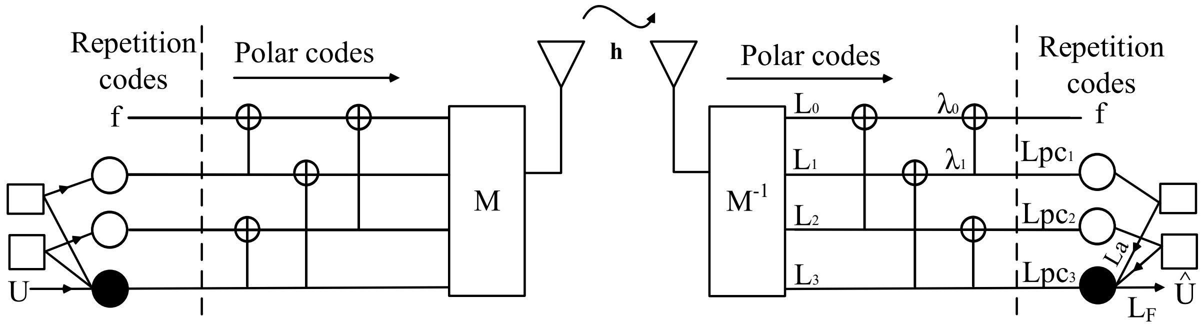

Fig. 4. An example of constructing the repetition concatenated Polar codes with 4 bits and f being the frozen bit.

M^{-1} and decoder C^{-1}. The *a priori* LLR for the successive cancellation decoding (SC) is defined as

$$L_A = \log \frac{P(x=+1)}{P(x=-1)} = \frac{2}{\sigma^2} \cdot \hat{y}, \quad (10)$$

with $\hat{y}$ being the output of demapper $\mathbf{M^{-1}}$.

The decoding for 2×2 kernel of Polar decoder is shown in Fig. 3, where the LLRs from the antenna are L_0 and L_1. LLR λ_0 emanating from the XOR operation is expressed as

$$\lambda_0 = L_0 \boxplus L_1 \quad (11)$$
$$= 2 \times \tanh^{-1}(\tanh \frac{L_0}{2} \cdot \tanh \frac{L_1}{2}) \quad (12)$$
$$\approx \text{sign}(L_0) \cdot \text{sign}(L_1) \cdot \min\{|L_0|, |L_1|\}, \quad (13)$$

to obtain estimate bit $\hat{u}_0$ via a hard decision (except if u_0 is a frozen bit). On the other hand, the LLR for bit $\hat{u}_1$ is

$$\lambda_1 = (-1)^{\hat{u}_0} * L_0 + L_1. \quad (14)$$

The $\hat{u}_1$ is the hard decision bit obtained from λ_1. In this paper, we perform the further decoding, therefore λ_0 and λ_1 are kept. At the end of the decoding the LLR a bit of information is obtained by adding up the value of the LLR of repetition codes. The sum of LLR presented in Fig. 4 is expressed as

$$L_F = L_{pc_3} + \sum_{i}^{d_v-1} L_{a_i}, \quad (15)$$

which is the sum of all LLR entering to the repetition decoder. The final results are then obtained by converting L_F into bits 0 and 1 by using hard decision.

III. Results and Discussion

The results of in this paper are divided in to two parts. First, we present BER performances of the investigated repetition concatenated Polar codes to observe the characteristic of the error correction capability as new codes. Second, we evaluate the effectiveness of repetition concatenated Polar codes as new codes compared to the original Polar codes.

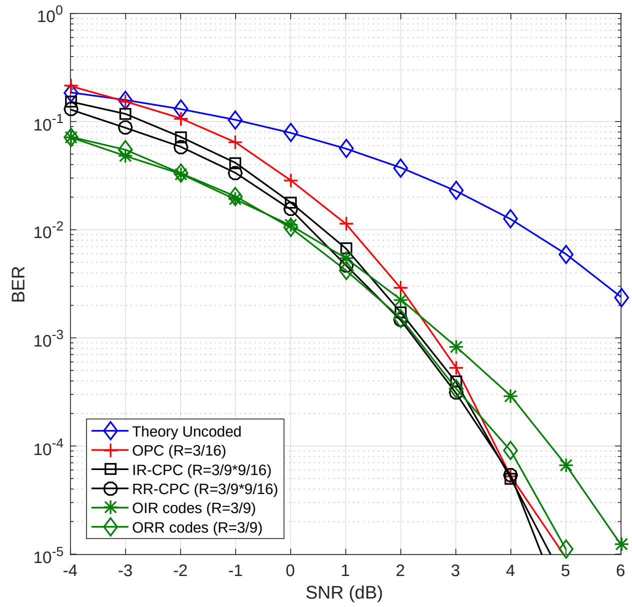

Fig. 5. The BER performances of IR-CPC and RR-CPC codes with their constituent elements under AWGN channels.

A. Performance of Repetition Concatenated Polar Codes and Their Components as New Coding Scheme

The performances of BER IR-CPC and RR-CPC codes compared to its constituent component is aimed to confirm that the combination of repetition codes and Polar codes increases the capability of the error correction of Polar codes. IR-CPC and RR-CPC codes in this paper have channels coding rate of $R = 3/16$ constructed from OPC $R = 3/16$ and original regular and irregular repetition codes $R = 3/9$. BER performances are evaluated under AWGN and Rayleigh fading channels.

Fig. 5 shows the performance of BER IR-CPC and RR-CPC codes under the AWGN channels, where theoretical BER of uncoded BPSK is shown as a reference. BER performance of RR-CPC and IR-CPC codes reach 10^{-4} at the SNR of 3 to 4 dB, while the constituent codes, OPC $R = 9/16$, original irregular repetition (OIR, and

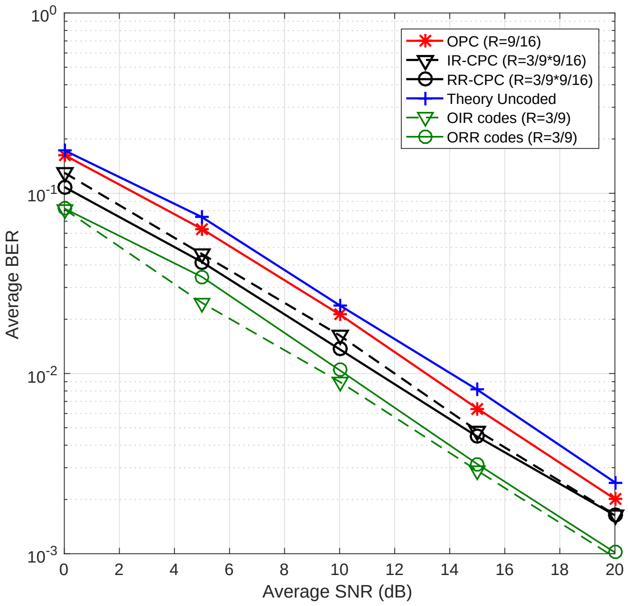

Fig. 6. The BER performances of IR-CPC and RR-CPC codes with their constituent elements under Rayleigh fading channels.

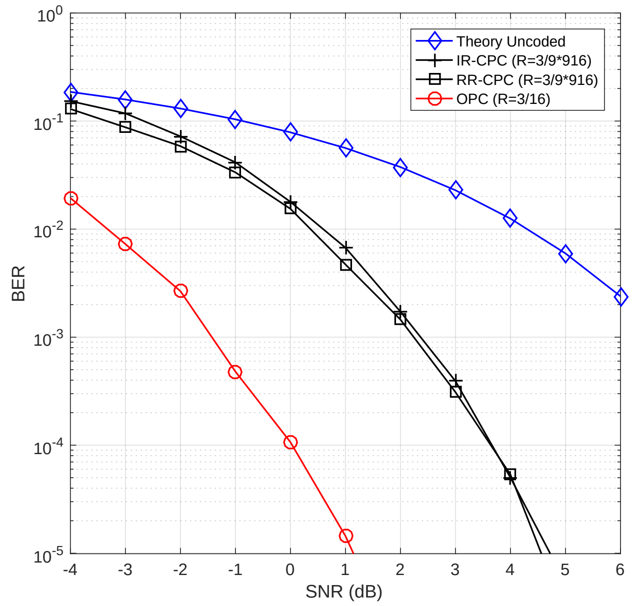

Fig. 7. The BER performances of IR-CPC and RR-CPC codes compared to that of the OPC under AWGN channels.

original regular repetition (ORR) codes $R = 3/9$ have worse performances.

These results confirmed that combination of Polar codes with repetition codes improve BER performances from its constituent codes. Fig. 5 confirms that the final BER performances are almost the combination of BER curves of the constituent codes. However, Polar codes have additional error correction capability from repetition codes as confirmed in Fig. 5 that Polar codes has worse performance only at low SNR region, but good performance at high SNR region.

Fig. 6 shows the BER performances of RR-CPC and IR-CPC under Rayleigh fading channels, where theoretical uncoded BPSK is used as a reference. The IR-CPC and RR-CPC codes has BER of 10^{-2} at SNR of 12 dB, while the constituent codes, i.e., Polar codes $R = 9/16$ has BER of 10^{-2} at SNR 13 dB; the original repetition has BER 10^{-3} at SNR 10 dB. This figure confirms that the combination of Polar codes with repetition, in one side increases BER performance when compared to Polar codes OPC $R = 9/16$, but in other side has worse performances compared to the original repetition codes $R = 3/9$. This happens probably because the location of frozen bits in the repetition concatenated Polar codes cannot adapt to the dynamic changes of Rayleigh fading channels.

B. Comparison of Repetition Concatenated Polar codes and Original Polar codes

Performances of repetition concatenated Polar codes with OPC is evaluated at the same rate. The comparison aims to obtain the best coding scheme, which is OPC or CPC codes. We evaluate the repetition concatenated Polar codes

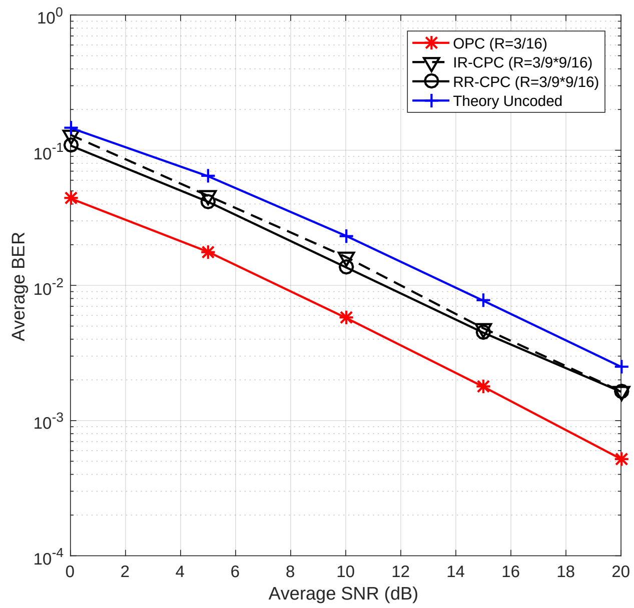

Fig. 8. The BER performances of IR-CPC and RR-CPC codes compared to that of the OPC under Rayleigh fading channels.

and OPC at a rate $R = 3/16$ under AWGN and Rayleigh fading channels.

Fig. 7 shows the BER performances of RR-CPC, IR-CPC codes, and OPC under AWGN channels, where theoretical uncoded BPSK is shown as a reference. The BER of OPC is 10^{-4} at SNR of 0 dB, while the BERs of IR-CPC and RR-CPC codes are 10^{-4} at the SNR of 3.5 dB. These results indicate that the capability of IR-CPC and RR-CPC codes is still weak compared to the OPC for the same channel coding rate. This fact may also indicate that the frozen bit of OPC is strong compared to the encoded bit protected by the repetition codes.

Fig. 8 shows the BER performances of RR-CPC and IR-CPC codes compared to the OPC in the Rayleigh fading channels, where the theoretical uncoded BPSK is shown as the reference. The original Polar codes has BER of 10^{-2} at SNR of 7.5 dB, while the IR-CPC and RR-CPC codes have BER of 10^{-3} at the SNR beyond 12 dB. The results indicate that the error correction capability of the RR-CPC and IR-CPC codes are still worse in Rayleigh fading channel compared to that of OPC at the same channel coding rate. This result indicates that the frozen bit of the OPC is strong compared to bit protected the repetition codes.

IV. Conclusion

This paper has studied the capability of error correction codes of OPC and CPC to evaluate the contribution of frozen bits for error correction. The evaluation was performed using a series of computer simulations for short Polar codes with a block length of 16 bits. This paper has provided results in terms of BER performances for IR-CPC and RR-CPC under AWGN and frequency-flat Rayleigh fading channels. This paper found that the OPC with traditional frozen bits are better than the IR-CPC and RR-CPC codes with less frozen bits even the information have been protected by the outer codes. These results indicates for short block-length that the frozen bits of OPC is powerful and may not be replaced by other bits, although the bits are protected by other outer codes resulting in concatenated Polar codes. The results of this paper are expected to be a reference in the future development of Polar codes considering the modification of frozen bits for better performances.

V. Acknowledgment

This research is supported in part by the ASEAN IVO project 2019–2021.

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